Mathematics for Elementary Educators—Multiplication of Fractions
See all handouts attached

Objectives:	I can use an area model to explain the multiplication algorithm.
Grade Level or Course Name	Mathematics for Elementary Educators
Estimated Time	75 minutes (30 minute modified plan for interview)
Pre-requisite Knowledge	Multiplication of whole numbers and fractions. Addition of whole numbers and fractions.
Vocabulary	Area Model, Distance = Rate times Time
Materials Needed	Problem of the Day sheet, Four student samples sheet, Assessment sheet
Iowa Common Core Content Standards	CCSS.6.PR: Understand ratio concepts and use ratio reasoning to solve problems. CCSS.5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.

Launch (How will you engage students in the content for the day?)
Introduce students to the POD:

> During my lunch hour I usually walk $\frac{3}{4}$ a mile in 15 minutes. Today it started raining 10 minutes into my walk and was unable to finish. How many miles did I walk instead? Explain how you arrived at your conclusion. Does your answer make sense? Why or why not?

Explore (How will students explore the content for the day?)

- Allow students ample time to explore the POD individually. Once students have a grasp on the problem have them share with a partner what they found and their justification.
- As students share, listen to conversations and look for different ideas to have students share. Ask students to share with the whole class.
- Ask the class how many students used multiplication to conclude ½ mile. How many drew a picture? How many used an area model?
- Introduce the four different student solutions. Have students work with their partner and fill out the second column.
- Students should share what they found with the whole class. Point out that student D has the best model for understanding the algorithm for multiplying fractions as the shaded area represents the numerator product and the denominator is represented by the shaded and non-shaded pieces representing the whole.

- Be sure to point out that Student A's model works well for this situation but would not work as well with 2/3 x 4/5. Why?
- Introduce the idea of $d = rt$ and ask how this idea relates to this problem. Challenge students to determine the rate. This should result in a rate of 3 mi/hr. Write the equation $d = 3t$ and ask students what the t is for this particular case (1/6). Discuss how to find 1/6 of 3 or 3 groups of 1/6.
- Hand out the final problem and explain that a student used a graph to show the product modeling the area idea presented by Student D. Have students work with their partner to determine what went wrong.
- Facilitate the discussion on what to do with the area model when one or both of the factors are greater than one.
- Introduce the idea of using the distributive property to accomplish this task. Pose the question, "Does the distributive property work for $1\frac{2}{3} \times 3\frac{2}{7}$?"
- Facilitate discussion. Be sure to point out the use of "and" and its meaning in mathematics.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
Ask students to explain how the area model works with fractions. Specifically ask students to find a product that can be represented by using Student A's method and Student D's method.

<u>Extension(s)</u>
Number line activity.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
- I will use thumbs up/side/down throughout both days of the lesson to check for understanding periodically.
- I will facilitate the explorations by asking guiding questions that both allow students to communicate what they understand and help them think about any misconceptions I notice in a different manner.
- Students will share several times to the whole class as well as communicate through writing with each other and the instructor. These forms of communication will serve as formative assessments for adjusting the lesson using appropriate scaffolding.

<u>Key Ideas</u>

Key ideas/important points	Teacher strategies/actions
Understanding the area model and how it applies to fractions.	Facilitate discussions that allow students to see how the area model works. Encourage students to investigate when the area model for Student D is the best case or when we arrive at a more reduced answer by using a model similar to Student A.

When a fraction is multiplied by a whole number *n* we are collecting *n* sets of the fraction which is repeated addition.	During the discussion about the rate problem, encourage students to think about how multiplication is simply an easier way to represent repetitive addition. How is that idea relevant to this situation?
Distributive property and how it applies to fractions as well as "FOIL".	Scaffold this idea using 12 x 15. This problem can be easily calculated using mental math by taking (10 x 15) and (2 x 15) and adding the two products together. How does this relate to fractions?

<u>Guiding Questions</u>

Good questions to ask	Possible student responses or actions	Possible teacher responses
How can multiplication aid in the solution of this problem? (Referring to the POD).	10/ 15 is the same a 2/3 so we need to find 2/3 of ¾. The word of in mathematics means to multiply.	What happens to the "threes" in the product? How is this evident referring to Student A's model?
Can we represent 2/3 x 4/5 in the same manner that Student A represented 2/3 x ¾ ?	It seems like we should be able to. I am having trouble because if I start with 2/3 I don't know how to take 4/5 of the 2 shaded.	Do you believe this is related to reducing? Can 2/3 x 4/5 be reduced? What model of the four models presented will work for this case? Will this model work for all cases?
Is it possible to find a product of two positive numbers such that the product is less than both numbers? Less than one of the numbers but greater than the other? Greater than both numbers?	The first two could be no or yes as students tend to struggle with both those ideas. The third will more than likely be yes as that is what they first learn when estimating products of whole numbers.	What about 2/3 of ¾? Which scenario does this product fit? Do you want to change your mind on any of the other answers? Give an example for all three cases.
When dealing with mixed numbers, how does the distributive property allow us to calculate products of fractions?	We can multiply the whole number by the whole number in the mixed number and the fraction in the mixed number. These two products can be added together to obtain the final product.	How does this work when we multiply two mixed numbers together? Provide an example and show how the distributive property aids in this product.

<u>Misconceptions, Errors, Trouble Spots</u>

Possible errors or trouble spots	**Teacher question/actions to resolve them**
Students may not understand which representation of the four provided solutions best models the algorithm for multiplication of fractions.	Prompt students to discuss the basic multiplication algorithm for multiplication of fractions. Ask how the numerator relates to the shaded area and what the denominator relates to in the context of the problem.
Students may believe that any situation can be modeled in the manner that student A did.	Scaffold this to show it is a special case because the products can be written as a reduced fraction. Use an example that does not work to show what is meant by this. An example could be ½ of ¾.
Students may not understand why the $d = rt$ formula is graphed the way it is and why the area model fails in this case.	Discuss the idea that this is a constant rate so it would be graphed as a linear line. The rate is 3 mi/hr based on the scenario. Ask students guiding questions to point out the difference in this set up versus the previous one. (One is a part times a part and the other is greater than one times a part.)
Students may have a difficult time using the distributive property idea for multiplication of fractions.	Use more than one example to show how to perform this operation with multiplication of whole numbers first. Scaffold this by first multiplying a whole number times a mixed number and then a mixed number times a mixed number.

Part I:

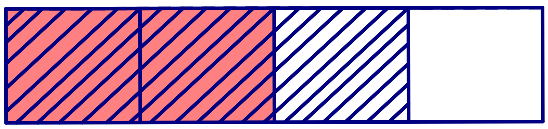
During my lunch hour I usually walk $\frac{3}{4}$ a mile in 15 minutes. Today it started raining 10 minutes into my walk and was unable to finish. How many miles did I walk instead? Explain how you arrived at your conclusion. Does your answer make sense? Why or why not?

Part II:

The following examples show four different student work samples. For each sample, determine what the student's thought process was and if it is correct model for this situation. If the thought process was not a correct model for this situation, adjust the model to make it accurate.

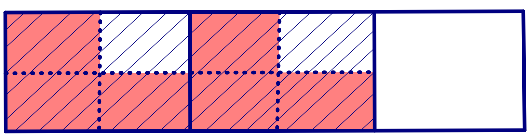
Explanations

Student A



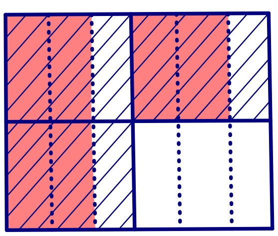
You would have walked $\frac{2}{4}$ of a mile.

Student B



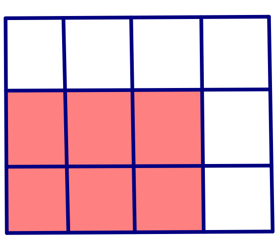
You would have walked $\frac{6}{9}$ of a mile.

Student C



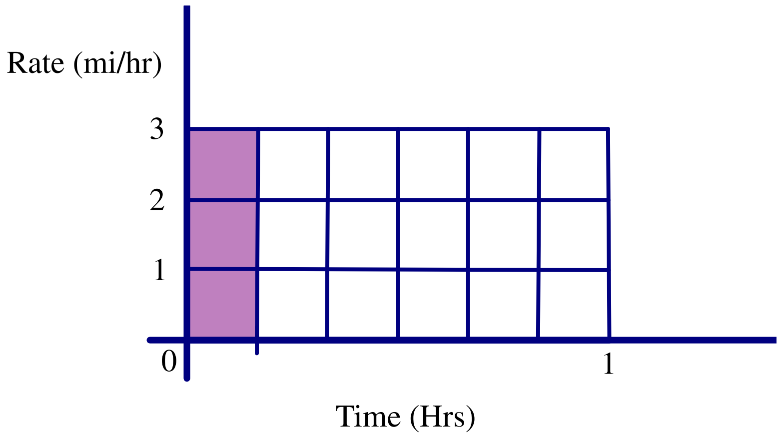
You would have walked $\frac{6}{12}$ of a mile.

Student D



You would have walked $\frac{6}{12}$ of a mile.

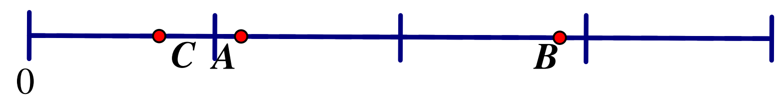
Part III:
Based on the area model from student D in the previous example, another student represented $D = 3t$ as a graph with rate versus time. This student is certain that the answer should be $\frac{3}{18}$. We found that the answer was $\frac{1}{2}$. How can this be?



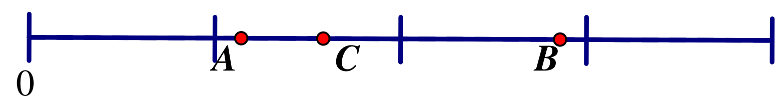
You would have walked $\frac{3}{18}$ of a mile.

Extension:

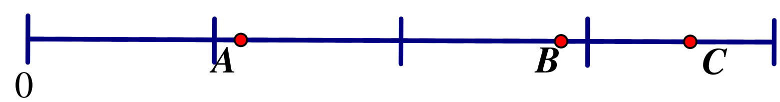
Given that *A* x *B* = *C*, determine a scale for the number line in each of the following cases. Explain in detail how you arrived at your conclusion.   Be prepared to share your answers with the whole class.



a.



b.



c.