Window Washing and Mathematics

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Algebra—(Arithmetic Sum and Sequence)

Objectives:	• I can convert a list of numbers into a function by making the whole numbers the inputs and the elements of the sequence the outputs. • I can write an explicit and/or recursive expression of a function to describe a real-world problem. • I can determine if a function is linear or exponential given a verbal description.
Grade Level or Course Name	Algebra
Estimated Time	90 minute class period
Pre-requisite Knowledge	Equation for a linear line, slope as a rate of change, writing a linear equation based on a given situation, recursive formula and explicit formula
Vocabulary	Summation, Gauss, Arithmetic mean
Materials Needed	TEAM sheet, Ticket out the door sheet, Graphing Calculator with Summation Capabilities or a computer with internet access to use Wolframalpha
Iowa Common Core Content Standards	• F-IF.3 Recognize that sequences are function, sometimes defined recursively, whose domain is a subset of the integers. • F-BF.1a Write a function that describes a relationship between two quantities. Part a: Determine an explicit expression, a recursive process, or steps for calculation from a context. • F-LE.1b Distinguish between situations that can be modeled with linear functions and with exponential functions. Part b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
Iowa Standards for Mathematical Practices	1. Making sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning.

<u>Launch</u> (How will you engage students in the content for the day?)
POD: You are at an interview for a job with two other interns. This job is for a window washing company that specializes in washing skyscraper windows. Part of your interview is a collaborative problem-solving task that you must complete with the two other interns. You are to create a bill for a skyscraper that is 14 stories tall and has 11 windows on each of the four sides of the building on every floor. The company charges $2 per window for the first floor, $3.50 per window for the second floor, $5 per window for the third, and continues in this fashion. The amount of money that will be charged for

this job must be accurate and accompanied by a clear explanation that the manager of the skyscraper will understand.

Explore (How will students explore the content for the day?)

1. Have students read the problem as they enter the room. Once students are familiar with the situation, have them get into their TEAMs of three.
2. In their TEAMs, students will create the cost for the job and type up a bill recording their process on the TEAM sheet as they go.
3. After students have successfully created the bill and provided an explanation, have TEAMs pick a resident expert. (A student who feels they understand well what their TEAM decided and can accurately communicate the TEAM's findings with others. This is a way to give students a sense of security. All students will be expected to share but students that are apprehensive have the opportunity to feel safe that they are not the experts and students that want to share feel that they get this opportunity. Again, every student shares with other groups.)
4. Once the resident expert in each group is selected (I usually give 30 seconds to pick one), have the non-resident experts move one and two groups over clockwise so that there are all new groups of three. Each resident expert will share their TEAM's bill with the two new group members first. Then each of the other group members will share their TEAM's bill. Students should defend their work if necessary but also make corrections where applicable. Students should list similarities and differences they see during this process.
5. Have students go back to their original TEAM and share with each other what they found. They should have met with a total of six different TEAMs through this activity. The purpose of the resident expert is to allow students to work out their own problems and assess their own misconceptions. Bouncing their ideas off of other students is a great way to confirm understanding and answer questions as well as see the problem through another lens.
6. As a whole class discuss the similarities and differences students noticed. Ask questions to help students see the connection to slope and rates of change. Have students do number 1 and 2 on the TEAM sheet.
7. Have students share what they found for number one and number two with their original TEAM. Review summation notation and have students determine how we could use summation notation in this situation connecting summations and the recursive formula found. Have students write a summation with their TEAM that fits the problem of the day.
8. Congratulate students and tell them that they have passed the first part of the interview. For the final task, the interviewee and the two interns must create a generic formula that the company can use for any number of windows and any number floors.
9. Each TEAM will prepare a short presentation to share on how they found their final formula. They will take turns sharing these at the end of class.

Summary/Close of the lesson

- I will ask guiding questions after the presentations that help students see similarities and differences in the summations. We will discuss how summations are not unique. If the different expected summations (see attached student work) are not all addressed during the presentations, I will help students see that these also work.
- As a ticket out the door, students will individually write what each part of their TEAM's formula means in terms of the problem. What does the slope represent? The y-intercept?

Extension(s)

- Have students write a spreadsheet program using a computer technology (Google drive spreadsheet, Microsoft Excel, whatever program is available in the district and students are familiar with) that verifies the recursive formula.
- Have students write a calculator program for the TI-84 Silver Plus edition calculator that asks for two inputs. The first will ask for the number of windows per floor and the second will ask for the number of floors. This program should have an output that tells the user how much they should budget for window cleaning of any skyscraper.

Homework

You are hired! Congratulations. The company offers you $11.25 per hour to start with $.50 annual raises. Assuming you work 40 hours per week, how much will you earn total in 6 years working for this company? In *n* years? Represent this situation using a summation.

Check for Understanding

- I will use thumbs up/side/down throughout the lesson to check for understanding periodically.
- I will facilitate the explorations by asking guiding questions with each TEAM that both allow students to communicate what they understand and help them think about any misconceptions I notice in a different manner.
- Students will share out as TEAMs several times as well as use the Ticket Out the Door. When students are defending their findings in their resident expert groups as well as their TEAM presentations I will be listening carefully assessing development of the mathematics for each group.
- I will assess student understanding when I collect the homework the next day.

Key Ideas

Key ideas/important points	Teacher strategies/actions
Writing a recursive formula	Help students see that the rate of change from one floor to another is constant by asking guiding questions. Ask students what this rate of change represents in the context of the problem. How can we obtain the next floor's cost if we know previous floor's cost?
Recognizing that this is a linear model	Through guiding questions, help students connect the constant change in cost in consecutive floors to a linear

	rate of change. Encourage them to graph the data points to see this change.
Creating an accurate model for the Problem of the Day	I will challenge TEAMs to defend how they obtained their final amount. I will facilitate the activity looking for misconceptions throughout the task and steer students in the right direction if they get too far off the path. Students will use the resident expert activity to confirm accuracy of their TEAM's bill.
Creating an accurate model for the generic formula	I will provide appropriate scaffolding to the problem to help students see the connections between formulas. When students present, I will facilitate class discussions and encourage TEAMs to edit their final formula if necessary.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
Can you make this process easier by using your calculator and its ability to make lists?	Yes if I use the y = feature and plug in an equation I can then create a table that would provide all the costs for each floor.	What equation would you use? Why? What would you start your table at? What would your change in x be? Why?
How is the cost of each floor related?	Each floor increases by $66 every time.	How does this relate to the change in the cost per floor and the number of windows?
How could we use the data collected to write an equation for each floor?	It's $66 difference per floor so we know that is our rate of change. However, we have to add $22.	If the rate of change is constant, what can we conclude?
Is there more than one rate of change in the situation?	Yes, the amount charged per window per floor is increasing at a rate $1.50 per floor.	How is this represented in the constant difference between consecutive floors?
What happens if we average the first and 14^{th} floors' costs? How is this related to the median of the numbers?	We obtain $517 for the average. This equals the median of the set of numbers and the mean.	Is there a way we could add these numbers together using this idea that would make the computations easier? What do we get when we average the 2^{nd} and 13^{th} floors' costs?

Misconceptions, Errors, Trouble Spots

Possible errors or trouble spots	Teacher question/actions to resolve them
Students are overwhelmed by all the	Ask students how we could organize this data.

computation that is necessary and keeping it organized.	How could a table help us? What necessary information should be included in the table?
Students have a difficult time creating the summation.	Ask students to look at their data to see how the data changes for every floor. How is this represented in the explicit formula? In the recursive formula?
When writing the generic formula for any number of windows and any number of floors, students may think that the slope is still 66 and/or the y-intercept is still 22.	Give students a different example. I have a building that is 5 stories tall and has 20 windows on each floor. Does their formula work if they calculate the sum using their summation and by creating a table.

Name _____
Date _____ Period _____

Team member names:

_____ _____ _____

Your Task:

You are at an interview for a job with two other interns. This job is for a window washing company that specializes in washing skyscraper windows. Part of your interview is a collaborative problem-solving task that you must complete with the two other interns. You are to create a bill for a skyscraper that is 14 stories tall and has 11 windows on each of the four sides of the building on every floor. The company charges $2 per window for the first floor, $3.50 per window for the second floor, $5 per window for the third, and continues in this fashion. The amount of money that will be charged for this job must be accurate and accompanied by a clear explanation that the manager of the skyscraper will understand. Record your thoughts on this paper as you go. The bill itself should be typed.

Please answer the following with complete sentences.

1. Can we find the 15^{th} floor's cost if we know the 14^{th} floor's cost? Can this situation be represented by a recursive formula? If so, find the formula and explain how you arrived at your formula. If not, explain why you believe this is true.

2. Can this same situation be represented by an explicit formula? If so, find the formula and explain how you arrived at your formula. If not, explain why you believe this is true.

Name _____
Date _____ Period ___

Ticket out the door:

1. Please write the formula that your TEAM found. Explain what each part of the formula means in terms of the problem. What does the slope represent? The y-intercept?

Homework:

2. You are hired! Congratulations. The company offers you $11.25 per hour to start with $.50 annual raises. Assuming you work 40 hours per week, how much will you earn total in 6 years working for this company? In *n* years? Represent this situation using a summation.

3. Find two different ways to calculate your average salary over the first six years.

Student Sample Solution:

Your Task:

You are at an interview for a job with two other interns. This job is for a window washing company that specializes in washing skyscraper windows. Part of your interview is a collaborative problem-solving task that you must complete with the two other interns. You are to create a bill for a skyscraper that is 14 stories tall and has 11 windows on each of the four sides of the building on every floor. The company charges $2 per window for the first floor, $3.50 per window for the second floor, $5 per window for the third, and continues in this fashion. The amount of money that will be charged for this job must be accurate and accompanied by a clear explanation that the manager of the skyscraper will understand. Record your thoughts on this paper as you go. The bill itself should be typed on a separate sheet of paper.

Sample Student Solution:
I know that there are 14 floors and 11 windows on each of the four sides. That means that there are 44 windows on each floor. We decided to find the amount for the first floor by taking $2 per window times 44 to get $88. Then we did the same thing for the next two floors and decided to make a table to organize and record the data.

1st Floor	2(44)	88
2nd Floor	3.5(44)	154
3rd Foor	5(44)	220
4th Floor	6.5(44)	286
5th Floor	8(44)	352
6th Floor	9.5(44)	418
7th Floor	11(44)	484
8th Floor	12.5(44)	550
9th Floor	14(44)	616
10th Floor	15.5(44)	682
11th Floor	17(44)	748
12th Floor	18.5(44)	814
13th Floor	20(44)	880
14th Floor	21.5(44)	946

We decided that we didn't want to actually do all the calculations to get the third column by typing in the cost per floor times 44 fourteen times. I noticed that the price per window increased by a $1.50 every time. Since this is a constant change and each amount will be times 44, I used my calculator to find all the totals for each floor. I went to the "y =" button. I typed in y = 44x. I then went to the "table set" screen and started my table at 2 and changed the delta x to 1.5. I had the calculator create a table of values and recorded the values in the third column of the chart. We then split up the work to add the 14 totals in the table by grouping the floors 1-5, 6-10, and 11-14. After we each calculated our totals we added those three numbers and got 7238. So the cost for cleaning all three windows is $7238. This amount surprised us. That is a lot of money to pay to wash the windows.

We wanted to check our answers and know that we are not allowed to check our work by just redoing the process we already tried. Our teacher always has us check our work by doing the problem in a different way. We discussed different options for this and came up with finding the average cost per window and multiplying that by the total number of windows. To find the average cost per window, we added all 14 costs together and divided by 14.

(2 + 3.5 + 5 + 6.5 + 8 + 9.5 + 11 + 12.5 + 14 + 15.5 + 17 + 18.5 + 20 + 21.5) / 14 = 11.75

So the average cost per window is $11.75. To find the total number of windows on the skyscraper, we took 44 times 14 and got 616 windows. So the total cost must be 616 times 11.75.

616 x 11.75 = $7238.

Since both answers agree, we feel we have the correct answer. We are ready to create the bill.

Bill for Washing Windows
The cost for washing the widows of your skyscraper was calculated as follows:

Floor	Cost per window	x Total Windows per floor	Cost per floor
1	$2.00	44	$88.00
2	$3.50	44	$154.00
3	$5.00	44	$220.00
4	$6.50	44	$286.00
5	$8.00	44	$352.00
6	$9.50	44	$418.00
7	$11.00	44	$484.00
8	$12.50	44	$550.00
9	$14.00	44	$616.00
10	$15.50	44	$682.00
11	$17.00	44	$748.00
12	$18.50	44	$814.00
13	$20.00	44	$880.00
14	$21.50	44	$946.00
		Total Cost	$7238.00

The total cost to clean all the windows for this skyscraper is $7238.00.

This amount was found by calculating the cost per floor for each of the fourteen floors. This cost per floor can be found by taking the 44 windows on each floor times the cost per window for that floor. Once all the floors have been individually calculated, the costs per floor were added together to obtain $7238.00.

Another way to calculate this bill would be to find the average cost per window. The average cost per window can be found by adding the fourteen individual costs per window together and dividing that sum by 14.

$(2 + 3.5 + 5 + 6.5 + 8 + 9.5 + 11 + 12.5 + 14 + 15.5 + 17 + 18.5 + 20 + 21.5) / 14 = 11.75$

Therefore, the average cost per window is $11.75. Each floor has 44 windows so 14 times 44 will give us the total number of windows.

$14 \times 44 = 616$

Taking 616 windows times 11.75 we obtain $7238.

Please pay the total amount due of $7238. Thank you!

Please answer the following with complete sentences.

1. Can we find the 15th floor's cost if we know the 14th floor's cost? Can this situation be represented by a recursive formula? If so, find the formula and explain how you arrived at your formula. If not, explain why you believe this is true.

Sample Student Solution:
Looking back at our table, I noticed that each floor's total cost increases by 66 every time. So I can find the 15th floor by adding 66 to the 14th floor's cost. So 946 + 66 = 1012 is the cost of the 15th floor. This can be written as a recursive formula since we know the starting point and that it increases by 66 every time. The formula would be

$$\begin{cases} a_1 = 88 \\ a_n = a_{n-1} + 66 \end{cases}.$$

2. Can this same situation be represented by an explicit formula? If so, find the formula and explain how you arrived at your formula. If not, explain why you believe this is true.

Sample Student Solution:
Since the values go up by the same amount every time it is linear. The 66 is the rate of change. So an equation can be written so that 66 is the slope.

y = 66 x + b

To find the b, I looked at the table and figured out what the cost of floor 0 would be by taking 88-66 to get 22. To find any floors total cost for that floor I can use the equation y = 66 x + 22. So the 15th floor's cost would be 66(15) + 22 = 1012 which is what I got in number 1.

Sample Student Solution for part 7 of lesson plan:
We know that we need to add the equation that finds the total cost of each floor 14 times. The summation would look like:
(66 x 1 + 22) + (66 x 2 + 22) … + (66 x 14 + 22).
We noticed that the 66 and 22 stay the same and that the x value from our explicit formula increases by 1 every time. After our class discussion about summations, we found the following summation would work:
$\sum_{i=1}^{14} 66i + 22$. This adds the equation for the total cost for each floor from 1 to 14.

Sample Student Solution for parts 8 and 9 of lesson plan:
We know that the summation has to start at floor 1 and go to n floors instead of 14. For the original problem knowing that there are 44 windows on each floor we could write this as $\sum_{i=1}^{n} 66i + 22$. However, we are not sure how to write the summation if we don't know how many windows are on each floor. We are stuck! We ask the teacher for help. The teacher asks us how the 66 and 22 relate to the 44 windows. We notice that 66 is one and a half times 44. The teacher asks us how this relates to the rate of change of the cost per

window per floor. The teacher then encourages us to look back at our first table. In looking at the first table, we notice that the 66 is the change in cost per window per floor times the total number of windows or 1.5(44) x where x is the floor we are on. However, we must account for what the 0th floor's cost is in the summation. This would be the number of windows divided by 2. Putting this all together we get 1.5(w)(x) + w/2 where w is the number of windows and x is the floor we are on. Writing this as a summation we get $\sum_{i=1}^{n} 1.5wi + 0.5w$ for the nth floor with w number of windows.

(Other forms of the summation include but are not limited to $\sum_{i=1}^{n} \frac{3}{2}wi + \frac{w}{2}$, $\sum_{i=1}^{n} 0.5(3wi + w)$, $\sum_{i=1}^{n} 0.5w(3i + 1)$, and $\frac{1}{2}\sum_{i=1}^{n} 3wi + w$. All the summations offer starting points for a rich discussion about what the different rates of changes are and how they vary with respect to one another. This includes both the change in price per window per floor as well as the rate of change in the price per floor.)

Ticket out the door:

1. Please write the formula that your TEAM found. Explain what each part of the formula means in terms of the problem. What does the slope represent? The y-intercept?

Sample Student Solution:

Our formula is $C = \sum_{i=1}^{n} \frac{3}{2}wi + \frac{w}{2}$ where C is the total cost to clean all the windows, w is the number of windows, and n is the number of stories in the skyscraper. The $\frac{3}{2}$ in the slope represents the rate of change in the cost per window from floor to floor. The slope $\frac{3}{2}w$ represents the rate of change from one floor to the next for total cost. The $\frac{w}{2}$ represents what the cost would be for the 0th floor if there was one but there is not one in this context. Based on the pattern, the cost per window on the 0th floor would be \$0.50 so this makes sense that it would be $\frac{w}{2}$ which is the same as $0.5w$.

Homework:

2. You are hired! Congratulations. The company offers you \$11.25 per hour to start with \$.50 annual raises. Assuming you work 40 hours per week, how much will you earn total in 6 years working for this company? In n years? Represent this situation using a summation.

Sample Student Solution:
I created a table again to organize the problem but I only did the first four years to get the basic idea. Since there are 52 weeks in a year, working 40 hours a week I would work 52(40) = 2080 hours a year.

Year 1	2080(11.25)	$23,400
Year 2	2080(11.75)	$24,440
Year 3	2080(12.25)	$25,480
Year 4	2080(12.75)	$26,520

The annual income increases $1040 each time. In my explicit formula, this would be the slope. Going backwards to year one, I get 23,400-1040 = 22360 for the y-intercept. The explicit equation is y = 1040x + 22360 where x is how many years I have worked for the company and y is the income for that year. To find the total income for the first six years I could use the summation: $\sum_{i=1}^{6} 1040i + 22360$. Using my graphing calculator, we get an answer of $156,000. The summation that represents the total amount earned after *n* years is $\sum_{i=1}^{n} 1040i + 22360$.

3. Find two different ways to calculate your average salary over the first six years.

Sample Student Solution:
From number one, I know that the total salary for six years is $156,000. To find the average, I would divide $156,000 by 6 to get $26,000. A second way to do this would be to find take the total made the first year and the total made the sixth year and average them. This will equal the average salary because the annual salary increases at a constant rate each year. Using the explicit formula I get $\frac{1040(1) + 22360 + 1040(6) + 22360}{2} = 26000$. Because both methods have the same answer, I know that the average salary over the first six years is $26,000.

Part II:

Throughout my teaching experience I have found that students often struggle with functions, specifically with recognizing sequences are functions. Seeing rates of change seems like a natural process to me; however, many of my students struggle with this idea as well. Knowing that these concepts are a struggle for students to master, I decided to focus on the *Functions* cluster of the Common Core State Standards (CCSS) for this lesson.

As a calculus teacher, I feel it is extremely important that students understand summations as they make calculating areas under a curve precise. If students can master writing a function that represents a recursive process, extend that into the explicit formula, and recognize how to write the summation for adding the first *n* terms of that sequence, they will be much more successful when calculating the limits of Riemann Sums. The main goal of the lesson I developed, the window washing problem, is the first "I can" statement in the objectives, "I can convert a list of numbers into a function by making the whole numbers the inputs and the elements of the sequence the outputs."

It should be assumed that students have been introduced to the three standards addressed and that this lesson will allow students to master the first standard while continuing to develop mastery of the other two. The three standards addressed by this lesson are as follows:

> F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-BF.1a Write a function that describes a relationship between two quantities. Part a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE.1b Distinguish between situations that can be modeled with linear functions and with exponential functions. Part b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

The window washing problem allows students to develop several equations that all model the same situation. Students build on the pattern discovered by indentifying quantities and their relationship in the problem. Both recursive relationships are used and explicit relationships in order to obtain the summation. The summation allows students to convert a list of numbers into a function with the whole numbers (each floor) as the input values and the price per floor as the output values.

Some students may choose to write the summation whose input values represent each floor but whose output values represent the cost of one window per floor. These students' summation would then be multiplied by forty-four in order to obtain the final cost. Not only do the total costs per floor increase linearly, the cost per window per floor also increases linearly. By writing the generic summation for n floors with w windows, students can see how the recursive formula and explicit formula are related. That is, the rate of change in total cost per floor is the same as the common difference in the recursive formula and the slope in the explicit formula, mastering the first standard and addressing the other two standards.

There are several linear relationships within the problem. Students will need to recognize that these relationships are indeed linear in order to write the recursive and

explicit formulas for the problem. They will need to make the connection between slopes and interpret what they mean in order to write a successful summation for both the generic formula and the homework problem. This problem also lends itself nicely to several Mathematical Practice Standards.

To execute an accurate solution to this problem, students will need to fully understand what the problem is asking as well as devise a plan for organizing the data per floor. Students will need to be sure that they have arrived at the correct amount for the bill and will need to check their answers making sure the answer makes sense with the problem. Students in my classes know that they are not allowed to check a problem by repeating a process. They understand that checking a solution is arriving at the same conclusion through a different lens. This allows students to practice the first Mathematical Practice Standard, "Making sense of problems and persevere in solving them" (National).

Students will need to make sense of the different rates of change and how they are represented in the each equation including the homework problem. Students will take a specific example with real values and write a generic formula in 3 variables to represent the most general case employing the second Mathematical Practice Standard, "Reason abstractly and quantitatively" (National).

Both the Taxi Cab video and the applet for calculating the amount of medicine in your system after so many doses inspired me to create a lesson that models a real world example with mathematics ("Using, WGBH). In the window washer problem students will use mathematics to communicate the total cost of the bill as well as the generic formula by creating tables, lists, sequences, and equations. The equations will show how

the quantities are related and which quantity depends on another. Students will model the problem using appropriate mathematics which is the heart of the Mathematical Practice Standard, "Model with mathematics" (National).

During the process, students may choose to use their graphing calculator to perform the repetitive computations by constructing a table as shown in the sample student work. Students may also ask if they can use Excel or create a spreadsheet in their Google Drive to collect the data allowing the computer program to perform the computations and keep track of the sum. It would be completely acceptable for students to utilize Woframalpha to compute the summation. Any student who chooses one or more of these options is practicing the Mathematical Practice Standard, "Use appropriate tools strategically" (National).

It is very important that students practice the Mathematical Practice Standard, "Attend to precision" as they will be critiquing their own work and others (National). Students will work in two different small groups cooperatively to problem-solve. In one small group (TEAM) students will work together to create the bill and generic cost equation. In the other group (resident expert) students will defend their solutions and critique the solutions of others. Students will check their work by using a different method for solving the problem.

Students will also practice the standard, "Look for and express regularity in repeated reasoning" due to the fact the entire problem is based on being able to express a pattern in a way that makes sense and is easy to use in calculating (National). In order to create the summation, students will need to see the repeated reasoning in the total per

floor and per window costs for each floor. They will need to connect the change in per window cost and change in per floor cost to the slope in the summation equation.

The window washer problem provides many opportunities for students to utilize different Mathematical Practices as well as master the function standard previously stated. The activity allows for students to communicate mathematics both orally and through writing. This problem allows students the opportunity to connect recursive equations and explicit formulas extending the connection to summations. It would be easy and natural to follow this lesson with a lesson on modeling a problem with an exponential function allowing mastery of the third CCSS.

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