Two years ago, the Boone Community School District adopted a new textbook series that is said to be aligned with the Common Core State Standards (CCSS). Having taught Geometry for the past six years at Boone High School, I was selected to be a member of the team that chose the new textbooks. Our team selected the Holt McDougal textbook series after being persuaded it was the solution to implementing the CCSS. While every section of the Holt McDougal textbook series has the CCSS stated as being "covered" by that section, I have not been convinced that students can become proficient in a standard simply by following the textbook instructions. It is as if the textbook publishers added a few questions, rearranged the ordering of topics, placed CCSS codes at the bottom of the first page of each section, and called the text aligned with the CCSS.

Hung-Hsi Wu confirmed my suspicions throughout his article "Phoenix Rising." He states, "…textbook developers are only slightly revising their texts before declaring them aligned with the CCSMS" (2). If textbook authors are simply going to slap a label on the binding of their slightly revised textbook that states, "Aligned with the Common Core State Standards," we will continue to produce students who are capable of playing the game of school mathematics instead of creating mathematicians and life-long learners.

Wu makes a good case that the CCSS are significantly different from what has driven our mathematics instruction for so long, the common mathematics textbook (1-2). He goes further to explain that textbook developers have yet to catch on that the CCSS are "radically different" from former standards (2). If students are to become mathematicians and life long learners, we have to drastically change the textbooks being used or throw them out all together.

Unpacking the CCSS and focusing on the Mathematical Practices, it is obvious that the textbook my district currently uses for geometry, *Geometry* by Ron Larson, "covers" the CCSS topics but ignores the Mathematical Practices. The text gives much away in its unnecessary scaffolding of problems. If textbooks drive current mathematics education and the developers of these textbooks are going to continue to ignore the true change, teachers will need to be trained to modify the curriculum in their hands to meet the needs addressed in the CCSS.

It seems that creating "patient problem-solvers," as Dan Meyer calls it in his video "Math Class Needs a Makeover", and the Mathematical Practices go hand in hand. Patient problem-solvers naturally perform all eight of the standards for Mathematical Practices. We need to give students opportunities to have delayed gratification in mathematics. The staircase problem as presented in the "Staircase Problem" video offered students exactly this kind of experience. Students left the classroom that day without an "answer," which in turn haunted them and provoked subconscious thoughts about the problem until the next day. When the problem was revisited the next day, it was seen in a new light. This process is what mathematicians do all the time, yet students are constantly being robbed of the opportunity to think like a mathematician when teachers/textbooks do not provide these experiences.

There doesn't seem to be a fix-all that implements the CCSS. Instead, textbook developers as well as teachers can implement the CCSS with fidelity by using several different techniques within the curriculum. To show how this can be done, I would like to explore five problems that can be found in the texbook, *Geometry* by Ron Larson. All five of these problems have been modified to allow students to think like mathematicians, perform like mathematicians, and encourage a growth mindset. With the adaptations made, all five problems aid in the development of the following Mathematical Practice standards:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

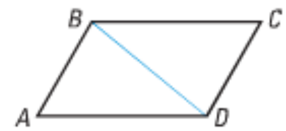
Further, all of the problems allow for several entry points which force students to make a plan, construct mathematical knowledge, and give meaning to the mathematics. The problems require students to make conjectures, then test theses conjectures, and finally formalize their findings. Students will not be handed information that they are capable of producing. Instead, students will need to make their own diagrams and determine if using graph paper or a dynamic software program like Geogebra or Geometer's Sketchpad (GSP) would help in the exploration of the problem. Students will need to clearly communicate in a coherent mathematical fashion with one another. The other Mathematical Practice Standards are not necessarily practiced in all the problems and will be addressed in relevant problems.

For the first problem, I would like to explore the idea that Dan Meyer suggests in his video - that textbooks often give too much information. We need to strip this extra information and only introduce it when appropriate scaffolding is needed. For example, the following problems found on page 520 of *Geometry* by Larson refer to theorems that are stated in the textbook, set up the proofs for the theorems, and give a picture with a hint on number 42 to draw an auxiliary line segment that is actually already drawn in the picture. It seems to be in the best interest of students as well as the CCSS to have students make their own conjectures about opposite and adjacent angles in a parallelogram based on the definition.

42. PROVING THEOREM 8.4 Use the diagram of quadrilateral *ABCD* with the auxiliary line segment drawn to write a two-column proof of Theorem 8.4.

GIVEN ▶ *ABCD* is a parallelogram.

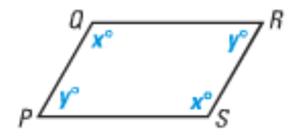
PROVE ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$



43. PROVING THEOREM 8.5 Use properties of parallel lines to prove Theorem 8.5.

GIVEN ▶ *PQRS* is a parallelogram.

PROVE ▶ $x^\circ + y^\circ = 180^\circ$

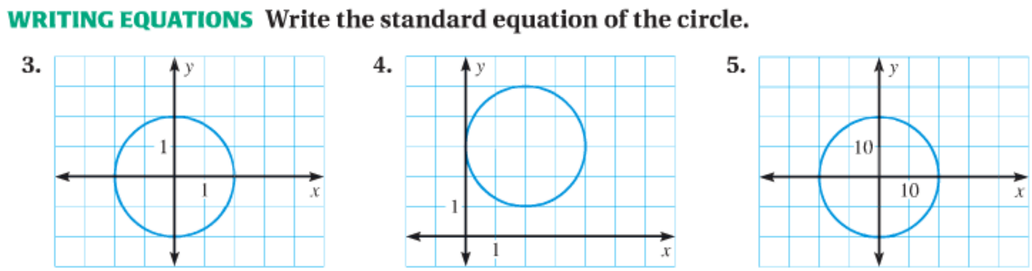


To execute this task in the fashion that Dan Meyer presents, students should be given the definition of a parallelogram. The definition states that a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Students should be asked to determine what relationships exist among the angles in the parallelogram. This allows students to make a conjecture about any and all relationships they believe exist. With a team, students should discuss possible conjectures and determine appropriate tools needed to test these conjectures and record their findings. This could include graph paper, a compass, a ruler, a protractor, or GSP. Once students have tested the conjectures, proofs should be written for the conjectures that students believe are true.

The role of the teacher becomes quite different when questions are asked in this manner. The teacher is responsible for asking guiding questions to help students arrive at the two desired conjectures. Students may find other relationships. For example, "the angle sum for a parallelogram is 360°" may be a conjecture if this fact has not already been discovered. It would be completely acceptable to write a proof for this conjecture; however, it is the teacher's role to make sure that the other two conjectures are tested and proven as well.

As students are working through the proofs, the opposite angles proof may be difficult to start if students do not add an auxiliary line. Instead of the teacher or the textbook telling students to do this, the teacher can ask guiding questions about congruent triangles and how they may aid in this proof. This allows students the opportunity to practice the seventh Mathematical Practice Standard, "Look for and make use of structure." Within this standard, it is explicitly stated that students can "…use the strategy of drawing an auxiliary line for solving problems" (National 8). When students are provided the opportunity to discover and prove these theorems, they continue to develop the standard CCSS-G-CO.11, "Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals" (National 76).

Allowing students the opportunity to discover the mathematics by stripping away extra information is one of many ways that teachers and textbook publishers can help create patient problem-solvers who think more like mathematicians. Often times, students are given math problems that they are fully capable of discovering on their own and then are asked to perform low order thinking by copying a process. This is very evident in the following problems found on page 702 in *Geometry* by Larson:



Students are handed the standard equation for a circle then asked to simply find the *h*, *k*, and *r*. These values are then substituted into the standard equation for a circle without any understanding that this equation is derived from the Pythagorean Theorem. Instead of presenting the equation for a circle in this way, omit giving students the equation of a circle. Pose the question, "How many points exist that are 5 units away from the origin on a coordinate plane?" Students should be asked to describe all of these points and note any patterns present. By allowing students to explore the problem this way allows for yet another Mathematical Practice Standard - standard 8, which states, "Look for and express regularity in repeated reasoning" (National 8).

Once the pattern is discovered, students should be asked how many lattice points could be found that fit this description. If students need more prompting to see the connection to the Pythagorean Theorem, guided questions should be asked to see how these points relate to the Pythagorean Theorem. This will help students make connections to Pythagorean Triples and the lattice points. After students have discovered that there are infinite points fitting this description that make a circle with center (0,0) and have derived the formula $x^2 + y^2 = r^2$, ask them to translate the circle 3 units right and 4 units up. Again, by asking guiding questions, the correct scaffolding can be used based on the needs of the individual students in the class. Students should generalize their findings and justify with a proof.

The original problem does not address any of the Mathematical Practice Standards. When the problem is rewritten so that students are allowed to discover the formula for a circle, it allows students to practice almost all of the Mathematical Practice Standards. The manner in which the textbook publishers address the equation for a circle completely undermines the following standard: CCSS-GPE.1 "Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation" (National 78). Rewriting this problem will allow a place for students to gain proficiency in the first part of the standard. Then extensions can be made to allow students to connect completing the square to the standard equation of a circle.

This brings me to my next point. When implementing the CCSS, I believe there are three necessary levels for each standard: introduction, proficiency, and mastery. When designing a task for students, it is necessary to know which of these three levels is the intention of the task and if there is room for multiple levels within a task.

As a current geometry teacher, I have spent much time specifically dissecting the geometry portion of the CCSS. In past practices in my district, transformations were void in the geometry curriculum. Instead, transformations were "covered" in depth in Algebra II and Pre-Calculus. This troubled me when I looked at two standards specifically in the *Congruence Cluster,* "Experiment with Transformations in the Plane." Standard number three states, "Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself" (National 76). Standard number five states, "Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another" (National 76). Searching *Geometry* by Larson, I found only one problem that addressed either of these standards on page 278.

40. ★ **OPEN-ENDED MATH** Some words reflect onto themselves through a vertical line of reflection. An example is shown.

a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word.

b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word.

This fact disturbed me as this problem barely introduces students to the idea of carrying a figure onto itself, let alone allows students to master these standards. Although the textbook publishers added an entire chapter to the textbook on transformations, the idea behind this particular problem is a shallow attempt at addressing these two standards. The following adaptation to this question allows students to be introduced to the idea of a figure carrying onto itself; however, the extension questions included allow students to become proficient in the standard.

In order to encourage the students to start thinking about how to carry a figure onto itself, it makes sense to start with the idea of regular polygons because of the multiple symmetries that each has allowing for multiple starting points. Introducing this concept with the following question will allow students to explore and make conjectures about reflections:

> Given the line of $y = x$ as a line of reflection, construct a geometric shape that maps back to itself.

Students should be encouraged to find as many geometric shapes as possible until the connection is discovered that any regular polygon will map back to itself as long as it is drawn so that the line of $y = x$ is a line of symmetry. If a student draws a square where the line of symmetry is a diagonal of the square, students should be encouraged to find another way to map the square onto itself without using a diagonal and vice versa. After students have a concrete foundation for this idea with reflections, they should be asked to find a polygon that carries onto itself with a rotation about the origin of 45°, 60°, 90°, and 120°.

Extensions can be made by asking students to determine the smallest possible rotation that carries each regular polygon onto itself. Students can make conjectures about this and can test these conjectures using Geogebra or GSP to arrive at a general formula. Students now have the opportunity to see that any rotation that is a multiple of an interior angle's supplement will carry a regular polygon onto itself. Once students have a good feel for finding all rotations that carry an image onto itself, allow students to construct regular polygons using rotations. Students will find that using GSP or Geogebra for this makes the constructions very simple.

For example, to construct a regular pentagon, students could take a point and rotate it about another point 72°, 144°, 216°, and 288°. These images would be the four other vertices needed to make a regular pentagon with the original pre-image point. Students have to derive the 72° from their previous findings and use the mathematics they discovered to find the multiples that carry a pentagon onto itself. Delving even deeper into this idea, students could explore compositions of reflections and rotations that carry an image onto itself. Throughout this task, students are constantly looking for structure and repeated reasoning.

Thus far, three examples have been given that show how to adapt problems to increase the opportunities for Mathematical Practices, as well as how to implement the

CCSS. The first consists of stripping away directions within a problem. The second involves rewriting problems so that students can discover the mathematics rather than reproduce a process. The third requires recreating tasks that include the different levels of introduction, proficiency, and mastery of a standard. Another way that teachers and textbook publishers can aid in the drastic change the CCSS brings with it is by spiraling the mathematics, which helps students see connections that exist within mathematics.

The following problems found on page 703 of *Geometry* by Larson are a way for students to connect algebra with geometry by writing equations of lines and segments that have certain attributes to the circles defined by the given equations. The problem solving involved increases just by rewriting the directions to ask students to specify if the given line has a relationship with the given circle and to state the relationship if one does exist. However, why stop there? This question could easily be taken to a whole new level by spiraling several topics within geometry and algebra and making it an open-ended question.

IDENTIFYING LINES Use the given equations to determine whether the line is a *tangent, secant, secant that contains a diameter,* or *none of these.*

31. Circle: $(x-4)^2+(y-3)^2=9$
Line: $y=-3x+6$

32. Circle: $(x+2)^2+(y-2)^2=16$
Line: $y=2x-4$

Right now, too much is given away and at the very least, the directions should be rewritten. Otherwise, students should be asked to find three equations that contain the three sides of a right triangle that circumscribes the circle with center (3,2) and has a radius of two. I tried to think of a problem that would encompass circles and tangent lines to the circle while still being within reach of my students. There are an infinite number of answers to this problem, allowing for more extensions and scaffolding to meet the needs of all learners. This problem will provide an opportunity for delayed gratification and help students practice patient problem-solving. Students should work with this on Geometer's Sketchpad or Geogebra, recording their trials with successes and errors. The issue with a problem like this is that as a teacher I have to be comfortable with students trying methods I haven't necessarily anticipated. Guiding questions must be well thought out so that they encourage students without giving away too much.

When students start this problem, I believe most will find an obvious location for the right angle of their right triangle to be (1,0) as in diagram 1. However, this still leaves a lot of room for exploration with finding the hypotenuse. There are many ways to increase the rigor on this question depending on the level at which the students are. If students struggle to find a hypotenuse that works in this case, simply asking a question that helps recall the relationship of a tangent to a diameter would be very helpful without giving away too much information.

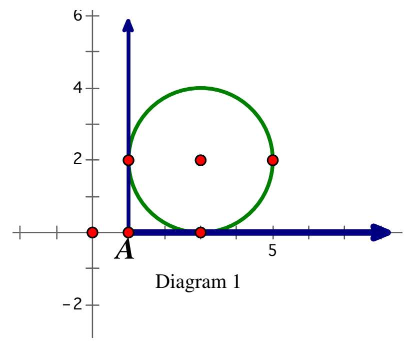
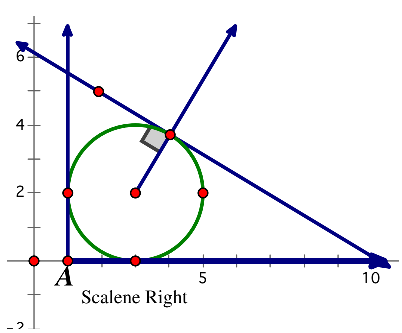
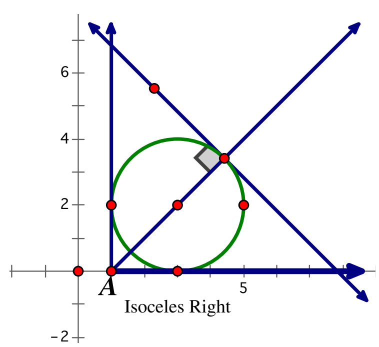


Diagram 1

Once students have successfully found a hypotenuse, a challenge could be posed by asking students to find a right isosceles triangle and a right scalene triangle that circumscribe the circle. (See both examples below.)



Scalene Right



Isoceles Right

Taking this question even further, students could be asked to describe all the locations that the point of tangency of the hypotenuse to the circle could be located to construct a circumscribed right triangle about the circle with the right angle at the point (1, 0). Remember in the beginning of this question students were not given the equation for the circle. This allows an opportunity to produce one and give the restriction on the domain that $3 < x < 5$ justifying the restriction.

Furthermore, the extensions can be even deeper. Students should be challenged to find a right triangle where the right angle is not located at the point (1, 0). This could lead students to find the other three lattice points that work as well as lead them to the discovery that any point on the circle $(x-3)^2 + (y-2)^2 = 8$ would work as the vertex for the right angle. (See diagram 4). Students could justify this using power of a point.

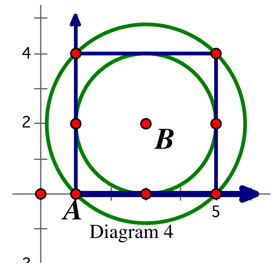


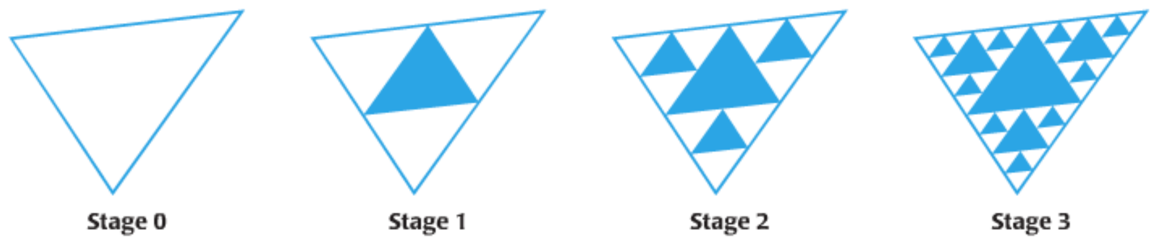
Diagram 4

Once again, changing the question so as not to give away too much allows for most of the Mathematical Practice Standards to be implemented as well as allow for introduction, proficiency, and possible mastery of the following standards:

- CCSS-GC.2. "Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle." (National 77).
- CCSS-GC.3. "Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle." (National 77).
- CCSS-GC.4. (+) "Construct a tangent line from a point outside a given circle to the circle." (National 77).

Sometimes the problems in the textbook do provide many opportunities for problem solving and creating patient problem-solvers. For instance, the following problem found on page 301 in *Geometry* by Larson is a good example:

43. MULTI-STEP PROBLEM To create the design below, shade the triangle formed by the three midsegments of a triangle. Then repeat the process for each unshaded triangle. Let the perimeter of the original triangle be 1.



a. What is the perimeter of the triangle that is shaded in Stage 1?
b. What is the total perimeter of all the shaded triangles in Stage 2?
c. What is the total perimeter of all the shaded triangles in Stage 3?

Even so, this problem could be tweaked to develop many more connections within mathematics and allow for more of the Mathematical Practices to be addressed simply by stripping away the questions and asking, "What is the total perimeter of the shaded triangles at the 100th stage?" The staircase problem as seen in the "Staircase Problem" video influenced the restructuring of this problem. Depending on where the students are in their learning, this question can be taken all the way to introduction of summation notation and geometric series exploration.

Using the midsegment theorem and the fact that the perimeter of similar figures is proportional to the corresponding sides, students could work cooperatively to generate a pattern. The instructor will need to be prepared to prompt students but still leave them wanting more. This problem has many starting points and room for scaffolding. If students try to jump straight to the 100th figure, the teacher can suggest trying to find the perimeter for the second figure, third figure, and so on. If students arrive at the conclusion that they only need to know the 99^{th} figure to find the 100^{th}, the teacher can challenge this thought by asking how they would obtain the 99^{th} figure's perimeter.

Organizing the patterns within the summation for each stage could look something like this:

Stage 0: 0

Stage 1: $1\left(\frac{1}{2}\right)$

Stage 2: $1\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)$

Stage 3: $1\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+9\left(\frac{1}{8}\right)$

Stage 4: $1\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+9\left(\frac{1}{8}\right)+27\left(\frac{1}{16}\right)$

Students could then piece together that the common ratio of the numerator is three and the denominator is two. Using summation notation, students could communicate this pattern and utilize a graphing calculator or Wolframalpha to find the 100^{th} term. One way this summation could be written is $\sum_{i=1}^{100} \frac{3^{i-1}}{2^i}$.

The beauty of this problem is that, if executed correctly, it includes the Mathematical Practice Standards as well as two CCSS listed below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- CCSS-FLE.2 "Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)." (National 71).

- CCSS-A-SSE.4 "Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments." (National 64).

There are many ways textbook developers and teachers can offer students a more mathematical experience versus the current regurgitation methods and shallow problem-solving opportunities. The five problems presented here show several different strategies that teachers can use to allow students to experience mathematics in the way it is intended, regardless of whether or not textbook publishers are on board. It takes a great deal of time to create and explore these tasks which is why teacher supports need to be in place to aid in this development, both in pre-service training and in-service training.

For true change in mathematics education to take place, teachers and textbook publishers alike need to recognize the CCSS are drastically different from past standards and celebrate this difference by developing appropriate tasks and embracing the paradigm shift. This is the only way that mathematics will have true reform and the product of mathematics education will be patient problem-solvers and life-long learners who naturally practice all eight of the Mathematical Practice Standards - in other words, true mathematicians.

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