Taking the Teacher out of the Student – Centered Lesson

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Part I: Revised Lesson Plan

Geometry—(9-10)—(Circles)
See all handouts attached

Objectives/I Can Statement:	I can identify central angles and inscribed angles. I can describe the relationship between an inscribed angle and the arc it intercepts. I can recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle.
Grade Level or Course Name	9-10 Geometry
Estimated Time	one - 90 minute class period
Pre-requisite Knowledge	• The measure of a minor arc is equal to the measure of its central angle. • An intercepted arc is the arc formed when segments intersect portions of a circle and create arcs. • Congruent triangles and similar triangles. • How to construct an inscribed triangle. • Diagonals of rectangles and squares are congruent.
Vocabulary	Central Angle, Inscribed Angle, Diameter, Radius, Chord, Circle Right Angle, Diagonals, Inscribed Triangles
Materials Needed	Computers with GSP available, Exit Ticket, Activity Sheet with homework, Compasses, Protractors, rulers, and sheets of paper
Iowa Common Core Content Standards	CCSS.G-C.2: Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Iowa Standards for Mathematical Practices	1. Making sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure.

Launch (How will you engage students in the content for the day?)

Students will be given the following scenario:

> A student came into class the other day excited about a discovery. The student thinks that it is possible to inscribe a pair of congruent triangles in the same circle. I haven't had time to determine if this is true or not so I am going to employ all of you to help. Do you think it is possible to inscribe a pair of congruent triangles in the same circle? Justify your thoughts.

Explore (How will students explore the content for the day?)

The content for the day will be split into two parts labeled A and B.

Part A:

1. Have students first start by working for three minutes on their own just sketching figures on a piece of paper to wrap their heads around the idea.
2. After students have explored individually, have them get into TEAMs of three allowing TEAMs to use a computer and GSP to experiment. As they explore, facilitate TEAM discussions and ask guiding questions to help students make conjectures and test their conjectures.
3. Once students find a pair of congruent triangles that can be circumscribed in the same triangle, ask students to explain how they know the two triangles are congruent. This will lead them to make connections between the different chords, arcs, and angles.
4. Once all TEAMs have had the opportunity to justify that the two triangles are congruent, have students share their observations as a whole class and critique each other's explanations. During this process, facilitate discussions about how the chords are related to the angle measures specifically focusing on the angle that shares the same chord. This will lead to a discussion about the arc measure.
5. Next, ask students to determine how the angle sum of a triangle relates to the degree measure of a circle. What does this tell us about the inscribed angle measure and its relationship to its intercepted arc? In their TEAMs students should make a conjecture and test the conjecture.
6. Have students report their findings. As a class, formalize findings and write the theorems for the measure of an inscribed angle and the relationships to its intercepted arc and angles that intercept the same arc are congruent.

Part B:

1. Ask students if they can find a pair of congruent triangles that can be inscribed in the same semicircle. Students should work individually on this question for three minutes.
2. Once students have had the opportunity to explore the question individually, instruct them to share what they have discovered with their TEAMs. As a TEAM have students work with their ideas recording trials of successes and failures noting any patterns they see. Students should be allowed to explore using GSP.
3. When every TEAM has a pair of congruent triangles inscribed in a semi-circle, have the reporter from each TEAM go to the board and draw what their TEAM found. Instruct students to put down all writing utensils and study the different diagrams for 60 seconds noting any similarities and differences. At the end of 60 seconds, instruct students to write for 90 seconds about what they noticed. With their TEAM, have students share what they wrote.
4. As a class, write this theorem. Ask students the following: We have proven that all triangles can be inscribed in a circle. Can all quadrilaterals be inscribed in a circle? In TEAMs, have students explore this idea.
5. Once TEAMs have had a little time to explore, as a class list the special types of quadrilaterals. Assign each TEAM two different types of quadrilaterals to investigate paired as rectangle with kite, parallelogram with isosceles trapezoid, and square with trapezoid. (This gives all TEAMs the opportunity to review constructions using reflections and diagonals of special quadrilaterals.) Have students report their findings to the whole class.

6. For homework students will be asked to organize the class' findings and explain how to determine what types of quadrilaterals can be inscribed in a circle.

Summary/Close of the lesson (How will you close your lesson and bring student understanding to a close for the day?)

1. Students will be asked to answer the first three questions on the ticket out the door sheet. These will need to be checked by the instructor before they may do the 4th question. The 4th question asks students to explain to a student who was absent from class today what they discovered and how they discovered it as a ticket out the door.
2. Students will be asked to justify their findings about what types of quadrilaterals can be inscribed in a circle for homework.

Extension(s)

- Can you inscribe a pair of similar triangles in the same circle?
- An angle that is x° intercepts an arc of 2x°. Where does the vertex of the angle lie? What is the probability that a point selected at random on the circle is the vertex of the angle?
- Prove that an inscribed angle is half the measure of its intercepted arc.
- How would you describe to someone how to inscribe a 3-4-5 triangle in a circle?

Check for Understanding (How will you assess students throughout and at the end of the lesson?)

- I will use thumbs up/side/down throughout both days of the lesson to check for understanding periodically.
- I will facilitate the explorations by asking guiding questions that both allow students to communicate what they understand and help them think about any misconceptions I notice in a different manner.
- Students will share out as TEAMs several times as well as use the Exit Ticket and the homework sheet.

Key Ideas

Key ideas/important points	Teacher strategies/actions
The measure of an inscribed angle is equal to half its intercepted arc.	Facilitate discussions within TEAMs that allow students to see that an inscribed angle is half the measure of the intercepted arc. Encourage students to think about the relationship between the angle sum of a triangle and the total degree measure of a full circle.
Two inscribed angles that intercept the same arc are congruent.	Ask guiding questions that allow students to see in their congruent triangles that the corresponding angles intercept the same arcs.
A triangle inscribed in a semicircle is a right triangle.	Encourage students and give them time to see the relationship between all the pairs of congruent triangles inscribed in a semicircle.

If a quadrilateral can be inscribed in a circle then opposite angles must be supplementary.	Help students make connections to what they found in Part A. Ask how the congruent triangles were constructed so that they could be inscribed and how this relates to quadrilaterals.

<u>Guiding Questions</u> (focus on the mathematics and using open-ended questioning)

Good questions to ask	Possible student responses or actions	Possible teacher responses
What relationship did you notice when you found a pair of congruent triangles that can be circumscribed in the same circle?	**Response 1**: I noticed making a rectangle or square drawing a diagonal that it was a diameter of the circle. This results in two congruent triangles. **Response 2**: I noticed that both triangles have to share a chord in order to work.	**Response 1**: Do you believe that is the only way this works? Can you inscribe a pair of congruent triangles that do not use a diameter as a shared chord? **Response 2:** Have you thought about making a parallelogram of some sort to start with? How might this work? Will any parallelogram work or does it have to be a special parallelogram?
How do you know that your triangles are congruent? What is true about corresponding angles in congruent triangles?	Using GSP all the corresponding sides and all the corresponding angles are congruent so I know the triangles are congruent.	How does the angle sum of a triangle compare to the degree measure of a full circle? What does this tell us about the angle measure of an inscribed angle and its intercepted arc.
What is true about every triangle that was inscribed in a semicircle?	All of them had the diameter as a side. Or all of them are right triangles with the right angle on the circle.	What conjecture could you make? How could you prove this conjecture based on the theorems we found in Part A?
When you first constructed the congruent triangles you involved a rectangle (or a square). What are attributes of the angles on a rectangle (or a square) that allow you to circumscribe it?	All the angles are 90° so each angle intercepts a diameter.	Is that the only relationship the angles have? How are opposite angles related? What other quadrilaterals have this same relationship (opposite angles are supplementary)?
How could transformations help us find a pair of congruent triangles that are inscribed in the same circle?	I don't know if translations would work but reflections and rotations might help.	How do you believe they would help? How could you take any pair of congruent triangles and transform them so that they are inscribed in the same circle?

Now that you have found a pair of congruent triangles, what do you know must be true about congruent angles and the relationship to the chord they intercept?	For every pair of angles that are congruent, each one intercepts the congruent chords.	How are congruent chords related to their arcs? What can you conclude about arcs that congruent angles intercept? What can you conclude about angles that intercept the same arc?

Misconceptions, Errors, Trouble Spots

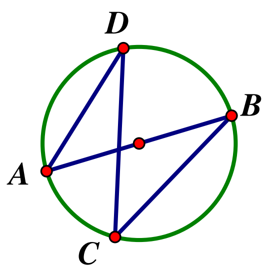
Possible errors or trouble spots	**Teacher question/actions to resolve them**
Students may find a counter example right away and assume that you cannot find a pair of congruent triangles that can be circumscribed by the same circle.	Ask students to think about all the different ways you can construct congruent triangles. If they struggle with this, ask students how they could easily construct congruent isosceles triangles from another shape? Or congruent right triangles?
Students may start with one triangle that is inscribed in a circle and have difficulty finding a second one.	Encourage students to use GSP to explore if they haven't done that yet. Using the inscribed triangle they have already, ask them how the second triangle might share a chord with it? How could this help with constructing a second triangle?
Students may believe that only rectangles can be inscribed in a circle.	Ask students to draw a circle. Have students pick four points on the circle and connect them to make a quadrilateral. If they choose four points that make a square or rectangle, encourage them to move one vertex so that it is no longer a right angle.
Students may not believe that some kites can be inscribed in a circle.	Ask students to think about how the diagonals of a kite are related and how this relationship could exist between two chords.

Name ______________________________
Date _______________ Period ______

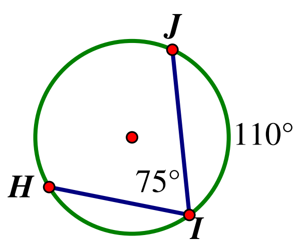
Exit Ticket

Please complete 1-3 and have them checked before completing number 4. Thank you!

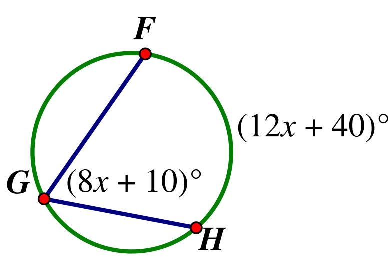
1. Given $\overline{AB}$ is a diameter, $m\widehat{BC} = 2x$, and $m\widehat{AC} = x$, Find $m\angle ADC$.



2. Find $m\widehat{HI}$.



3. Find $m\angle FGH$.



4. Please explain in detail what you discovered today and how you discovered it to a student who was absent.

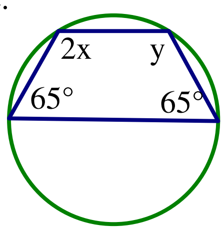
Name ______________________________
Date _______________ Period ______

Homework

1. Can a quadrilateral always be inscribed in a circle? Why or why not? What quadrilaterals can always be inscribed, what quadrilaterals can sometimes be inscribed, and what quadrilaterals can never be inscribed in a circle? Explain how you came to each conclusion.

Find the values of the variables for each. In number 3, explain how to find the value of *d* and *e* in two different ways.

2.



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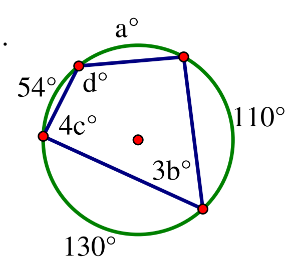
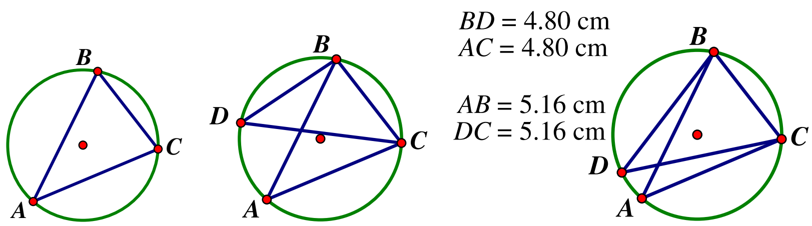


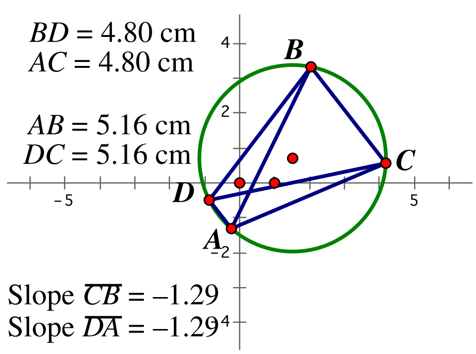
Sample Student Solution for Part A of Lesson:

Case 1 for launch question:

As a student, I could construct a circle and connect three points on the circle with segments to create inscribed triangle *ABC*. When we first started working with congruent triangles, they often shared a side. I would use this idea to construct a second triangle, triangle *ABD* that shared a side with triangle *ABC.* Manipulating the points *A* and *D* and using the measuring tool I would find a pair of congruent triangles. Triangle *ABC* is congruent to triangle *DCB* by SSS congruence postulate.

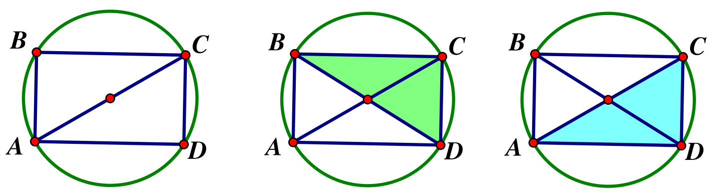


Because the corresponding sides are all congruent and the sides are chords in the same circle, it looks like angles that intercept congruent chords or the same chord are congruent. Looking at the diagram I drew, it looks as though quad *ADBC* is an isosceles trapezoid. Using the measuring tools, I could find the slopes of *AD* and *BC* concluding that they are indeed the same.



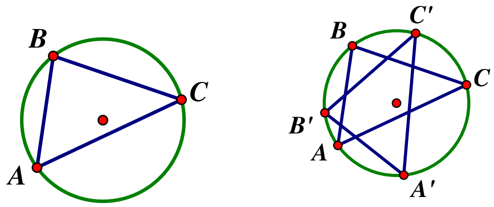
Case 2 for launch question:

As the student, I could start by constructing a rectangle and then circumscribe the rectangle. Using a diagonal of the rectangle, I would divide the rectangle into two congruent triangles. The diagonal is a diameter of the circle. The conclusion is that triangle *ABC* is congruent to triangle *CDA* by either SSS congruence postulate or by HL. I could easily construct another pair of congruent triangles by constructing diagonal *BD* and shading the two different triangles. In this case triangle *BCD* and triangle *ADC* are congruent. Because the corresponding sides are all congruent and the sides are chords in the same circle, it looks like angles that intercept congruent chords or the same chord are congruent.



Case 3 for launch Question:

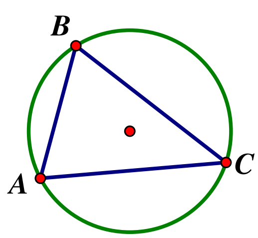
Because I know it is possible to circumscribe all triangles, as the student, I could decide to start with one triangle. Using GSP I would construct triangle *ABC* and then construct the circle that would circumscribe it. Knowing that rotations preserve distance, I would use the transformations tool to mark the center of the circle as the center of rotation. Rotating the triangle any degree other than 360° would create an image that is still inscribed in the circle and is congruent to triangle *ABC*. For example, I chose to rotate triangle *ABC* 60° to obtain triangle *A'B'C'*. These two triangles are congruent because rotations preserve distance so all corresponding sides are congruent.



Because the corresponding sides are all congruent and the sides are chords in the same circle, it looks like angles that intercept congruent chords are congruent.

Part A post launch question discussions (5-6 of lesson plan):

The angle sum of any triangle is 180° and the degree measure of a circle is 360°. Looking at an inscribed triangle in a circle it appears that the measures of the inscribed angle must be half the measure of the intercepted arc because the sum for the inscribed angles is half the sum of the full circle.

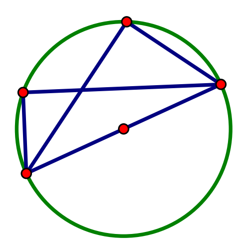


$m\angle A + m\angle B + m\angle C = 180°$ Angle Sum Theorem
$\overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CA} = 360°$
Because $\angle A$ intercepts $\overset{\frown}{BC}$, $\angle B$ intercepts $\overset{\frown}{AC}$, and $\angle C$ intercepts $\overset{\frown}{AB}$, then it makes sense that the angles are half the measure of their intercepted arc.

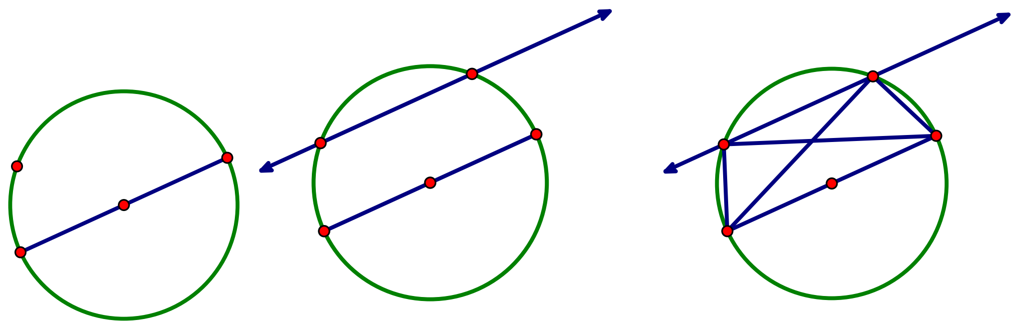
Using GSP and the measuring tool, this conjecture seems to hold true.

Sample Student Solution Part B of Lesson:

As the student, I would use GSP to construct a semicircle. An important observation at this point is that every triangle must have the diameter as a side. Constructing several triangles, I would find all of them to be right triangles. Because I know the hypotenuse is the same and all are right triangles, I needed to find two triangles that have a leg that is the same length guaranteeing these triangles are congruent due to HL. Thinking about the ideas from Part A, I would use an isosceles trapezoid and its

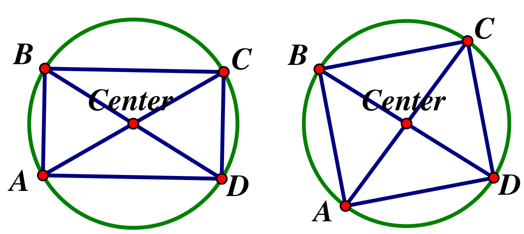


diagonals to construct a pair of congruent triangles. I would construct a semicircle and a point on the semicircle. Next, I would construct a line through the point and parallel to the diameter. The intersection of the parallel line and the circle should be marked as the corresponding vertex on the second triangle.

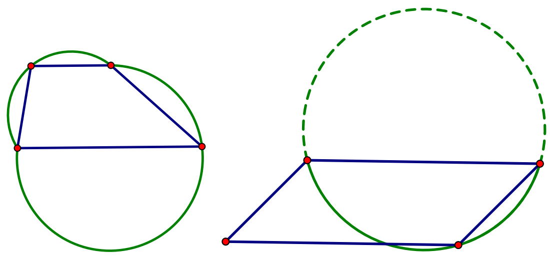


Part B continued (4-6 of lesson plan):

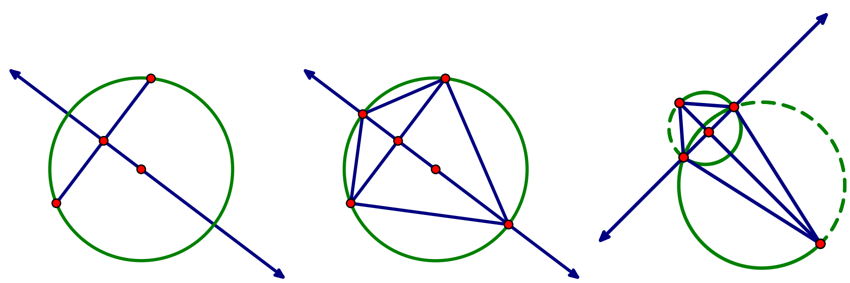
From the experiments we have done so far, it seems that rectangles, squares, and isosceles trapezoids can be inscribed in a circle. The question is, can they always be? What about parallelograms that are not rectangles? Can any trapezoid be inscribed? What about kites? Using GSP, as the student, I would investigate these questions finding that squares and rectangles can always be inscribed in a circle. Because their diagonals are congruent, let the intersection of the diagonals be the center of the circle and it works every time.



Working with trapezoids and starting with a pair of parallel chords in a circle having different lengths an isosceles trapezoid can always be constructed. Investigating the isosceles trapezoid I would find it works every time. However, when trying trapezoids that did not have congruent legs, I would find these trapezoids could not be circumscribed. The same issue would arise with non-rectangular parallelograms.



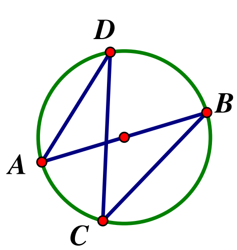
Starting with a circle, it is possible to always inscribe a kite. Knowing that the diagonals of a kite are perpendicular and one diagonal is bisected by the other, I could take any chord that was not a diameter and construct the perpendicular bisector of the chord. (This would be a diameter of the circle.) This construction always results in a kite, however, two of the angles of the kite always measure 90°. Trying a kite that did not have a pair of 90° angles was not successful.



Sample Student Solution for Exit Ticket:

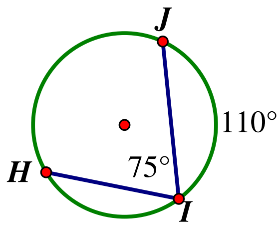
Please complete 1-3 and have them checked before completing number 4. Thank you!

1. Given $\overline{AB}$ is a diameter, $m\overset{\frown}{BC} = 2x$, and $m\overset{\frown}{AC} = x$, Find $m\angle ADC$.



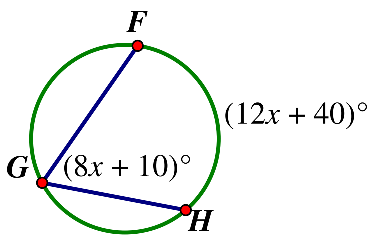
I know that the $m\angle ADC = m\angle ABC$. With the given information I can find $m\angle ABC$. Becasue $\overline{AB}$ is a diameter, $m\overset{\frown}{BC} + m\overset{\frown}{CB} = 180°$. Then $2x + x = 180°$ and $x = 60°$. Since $x = m\overset{\frown}{AC}$, $m\angle ABC = \frac{1}{2}x = 30°$. Thus $m\angle ADC = 30°$.

2. Find $m\,\widehat{HI}$.



Because $m\angle HIJ = 75°$ and it intercepts $\widehat{HJ}$ I know that $m\,\widehat{HJ}$ is 150°. Therefore $m\,\widehat{HI} = 360° - 150° - 110° = 100°$

3. Find $m\angle FGH$.



$2(8x + 10) = 12x + 40$
$16x + 20 = 12x + 40$
$4x = 20$
$x = 5$

$m\angle FGH = (8)(5) + 10 = 50°$

4. Please explain in detail what you discovered today and how you discovered it to a student who was absent.

Possible Student Solution:

Today I found that inscribed angle measures are related to the arc they intercept. The inscribed angle is half the measure of the intercepted arc. I also found that inscribed angles that intercept the same arc or congruent arcs in the same circle are congruent. I made these conjectures and tested them with my team. We started by trying to circumscribe a pair of congruent triangles. It was easier to start with a rectangle to do this but there are many ways it can be done. In our team we looked at the relationship between the angle sum of a triangle and the total degrees of the intercepted arcs. Since the angle sum is always 180° and the intercepted arc sum is always 360° we concluded that the inscribed angle must be half the measure of its intercepted arc. We have not proven this yet.

Sample Student Solution for Homework:

1. Can a quadrilateral always be inscribed in a circle? Why or why not? What quadrilaterals can always be inscribed, what quadrilaterals can sometimes be inscribed, and what quadrilaterals can never be inscribed in a circle? For all of the quadrilaterals that can always be inscribed, what relationship exists between opposite angles?

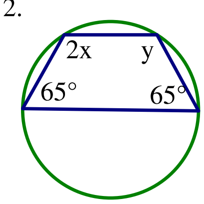
Possible Student Solution:

Not all quadrilaterals can be inscribed in a circle. We found as a class that all squares, rectangles, and isosceles trapezoids can always be inscribed in a circle. Squares and rectangles have congruent diameters and can be circumscribed easily by using the midpoint of the diagonals as the center of the circle. I assumed that any quadrilateral with four congruent angles could be inscribed. However, isosceles trapezoids will not have any right angles, yet they can be inscribed. We also found that some kites can be inscribed in a circle. For this to be true, the kite must have a pair of 90° angles. This is due to the diagonals of the kite consisting of a diameter and a chord that is not a diameter. Because a diameter is a diagonal, the angles that intercept the diameter will be 90°. We found that parallelograms that do not have 90° angles cannot be inscribed in a circle.

Putting all of this together, every time a quadrilateral could be inscribed in a circle, the opposite angles were always supplementary.

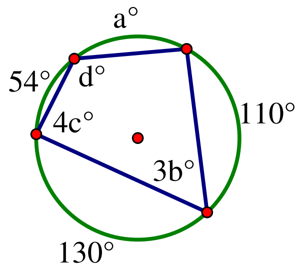
Find the values of the variables for each. On number 3, describe two different ways to find the measure of *d*.

2.



$$2x + 65 = 180° \qquad y + 65 = 180°$$
$$2x = 115 \qquad y = 115°$$
$$x = 57.5$$

3.



a° = 360° - 110° - 130° - 54° = 66°.

2(3b) = 54 + a	2(4c) = 110 + a
6b = 54 + 66	8c = 110 + 66
6b = 120	8c = 176
b = 20	c = 22

Therefore a = 66, b = 20, and c = 22.

The value of *d* can be found in two different ways. First $d + 3b = 180$ and so $d = 120°$. The other way is that *d* is half the measure of $110 + 130 = \frac{1}{2}(240) = 120°$.

Part II: Essay

As a current geometry teacher, I was very interested in the lesson about inscribed angles and their relationship to the intercepted arc. An activity that I have used in the past is one that I discovered on the Texas Instruments website designed to utilize the TI-84 graphing calculator. In looking at the provided lesson, I found that the activity that I have used in the past is basically the same as the provided lesson only constructions are made with the calculator instead of using a compass and protractor.

The original lesson falls under the full recipe for almost all of the TARP categories. It lacks multiple solution strategies. In fact, the manner in which it is presented appears to have only one solution strategy: follow the directions provided. Other than allowing students to make a conjecture after the exploration, analysis is absent. Because the activity gives so much away, there is really no need for revision. Either the students follow the directions or they don't. Even though students are asked to share their tables with each other, the original lesson is an ill attempt to have students present.

The following TARP rubric includes the ratings for the revised lesson in each category.

	Full Inquiry (Student Centered)	**Moderate Inquiry (Student and Teacher Centered)**	**Full Recipe (Teacher Centered)**
Task	Problems/Questions are student initiated; Tasks are couched in a context meaningful to students. Tasks are rich, supporting multiple solution strategies and genuine reflection.	Problems/Questions are initiated by teacher (or materials). Tasks are couched in a context meaningful to students. Tasks are rich, supporting multiple solution strategies and genuine reflection.	Problems/Questions are initiated by teacher (or materials). Tasks lack context and /or are not sufficiently rich to support multiple solution strategies or reflection.
Analysis	Techniques for analyzing problems are developed/initiated by students.	Techniques for analyzing problems are developed/initiated by teacher or by students with significant guidance.	Techniques for analyzing problems are developed/initiated by teacher (or materials).
Revision	Opportunities are provided to critically evaluate/revise questions and their solutions.	Opportunities are provided to critically evaluate/revise solutions to mathematical questions.	Revision opportunities are limited or altogether lacking.
Presentation	Students present findings to an audience beyond the teacher (e.g. peers, another classroom). Findings are reported in a variety of ways (e.g. presentation, poster, video).	Students present solutions in a format dictated by the materials to an audience beyond the classroom teacher (e.g. peers, another classroom).	Students present solutions in a format dictated by the materials to the classroom teacher.

Due to the multiple entry points and solution strategies, the revised lesson falls under the Moderate Inquiry for the Task category of the TARP rubric. The question presented to students that prompts the investigation is teacher-generated, which is why the revised lesson falls short of the Full Inquiry rating. With all the freedom offered by this problem, students will constantly need to revise and analyze their findings and strategies they are using. Therefore, Analysis receives a Full Inquiry rating.

Through discussions with their cooperative teams, students will continue to develop questions and make plans to test these questions. Even though the original question is prompted by the teacher, many other questions will arise while working in the teams. When students report their findings to each other, they will evaluate where they are in cognitive understanding of the problem and make appropriate adjustments and adaptations to the solution strategy until they arrive at a plausible answer. Consequently, the revised task falls under Full Inquiry for the Revision category.

The Presentation category was a struggle for me when designing this lesson. Because students have so much freedom with this activity, they may choose to report in several different ways; however, the lesson as it stands really only has students reporting to their classmates and absent students. I felt the best score for the Presentation category was Moderate Inquiry. Again, this could score in the Full Inquiry category when implemented if I, as the teacher, take advantage of sharing opportunities as they arise. While it is difficult to say exactly where the discussions will lead, through appropriate questioning by me, this lesson is rich in the connectedness it offers within the mathematics' "big ideas."

The original lesson states the intended curriculum is the standard CCSS.G-C.2: "Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle" (National). This standard involves deep connections among many attributes of circles. However, opportunities to employ these relationships have been ignored. The revised lesson I wrote allows students to make connections between chords, arcs, inscribed angles, congruent chords, and congruent inscribed triangles. By altering the original lesson, students are now able to see the connections between angles within an inscribed quadrilateral.

The revised lesson allows students opportunities to practice several of the Mathematical Practice Standards. The first standard states, “Make sense of problems and persevere in solving them” (National). There are many different strategies that students can use to work through this problem; however, to begin the process, students must make sense of the problem. Because there are so many different approaches to pursue, students have to make a plan that is based on prior knowledge and evaluate their work along the way. Students have to make connections between arcs, chords, and inscribed angles that intercept these arcs and chords. Additionally they need to make connections to congruent chords and arcs. Students must reason abstractly to accomplish this task, utilizing the standard “Reason abstractly and quantitatively” (National). Throughout the activity several conjectures will be made and justification will be given for these conjectures. Thus, the standard “Construct viable arguments and critique the reasoning of others” is practiced (National). The last Mathematical Practice Standard addressed is “Look for and make use of structure” because students will need to make connections to the patterns they see in working through the task in order to make viable arguments (National).

The revised lesson also incorporates some degree of all of the National Council of Teachers of Mathematics (NCTM) Standards:

- Problem Solving Standard
- Reasoning and Proof Standard
- Communication Standard
- Connections Standard
- Representation Standard

The design of the lesson is such that students are allowed multiple opportunities for problem solving. Students will use prior knowledge to develop new knowledge through connecting the two. Students will use many different strategies throughout the lesson to make conjectures, test these conjectures, and defend these conjectures. The nature of the lesson allows for much communication among students. Students will not only communicate orally with one another, but they will also analyze and evaluate the findings of other students as well as evaluate their own findings. Because students are transferring knowledge of congruent triangles and inscribed triangles, connections are being made to angle measure of inscribed angles and their intercepted chord and how this relates to its arc. Students will propose many different solutions to this problem and share these with each other in teams and with the whole class. This allows students to organize their thoughts and formalize their findings.

The revised lesson also provides several possibilities for formative assessments. As the facilitator in this activity, I will constantly be monitoring different teams and the mathematic ideas students are developing and implementing. Students will not be given answers to questions they ask; instead, I will ask them higher order thinking questions to help students answer their own questions while assessing any misconceptions at the same time. Students will be asked to share through mini-presentations to the whole class. This will allow me to assess each team midway through the lesson making sure that the mathematical foundation is strong. Students will apply their conjecture and have this application checked by me for immediate feedback. Students will self-assess through thumbs up/side/down; a technique I use to obtain a general idea in a few seconds of how well students feel they understand the material and process at certain points in the lesson.

There are two different problems where students are asked to summarize the material discovered in their own words. As most of the activity will be done in teams, this allows me to see individually where any misconceptions may lie as well as affirm that students have gained the intended knowledge. The first question asks students to explain to a student who was absent what was discovered for the day and how it was discovered in their own words. The second summary also has an extension where students have to formalize their team's findings through generalizations. This question not only asks students to summarize what was discovered about inscribing a quadrilateral but also asks students to determine whether a quadrilateral can be inscribed or not. Thus, students are not merely regurgitating the day's events on paper; they are continuing to build on the acquired knowledge.

This revision seems like a drastic overhaul. Restructuring the lesson with the Mathematical Practice Standards and the NCTM Standards in mind was a very time-consuming investment. However, implementing the revised lesson plan allows students more opportunities to think like mathematicians and practice mathematics in the way it was intended. In the original lesson plan students do not model mathematical thought; instead, they follow the mathematical thought process of the instructor. In the future, I will be re-assessing the activities I plan to ensure the implementation of the TARP rubric to guarantee that students are offered authentic mathematical experiences whenever they can.

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