Annie Carpenter
Unit on Properties of Circles

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Introduction

When students think of circles, most often, pies, pizzas, and cookies come to mind. Many times students think that the area formula and circumference formula are the only properties of circles relevant to mathematics. However, this is only a small representation of the many properties of circles and their everyday use.

As a secondary mathematics instructor at Boone High School, I am currently in my 5th year of teaching geometry. The geometry classes consist of mainly sophomore students but may have freshmen, juniors, and seniors enrolled as well. This fall, the Boone Community School District adopted new math textbooks including a geometry book by Larson titled *Geometry* that is more traditional in nature. Wanting to give my students a more progressive, problematic curriculum, I chose to write this unit incorporating what I have learned while taking this class. I purposely chose a unit on circles hoping to instill in students the many properties that circles have that go beyond the usual expectations of circumference and area.

One of the main goals for this unit is to allow students to make their own conjectures and then test those conjectures using appropriate manipulatives and/or technologies. Making conjectures is a natural part of being a mathematician which is often over looked. Many times in traditional curricula, the conjectures are already stated for the students. Mason, Burton, and Stacey believe, "Conjectures…form the backbone of mathematical thinking." (Mason, Burton, & Stacey, 2010, p. 59). In my own mathematical practices, I have found that the more I conjecture and test those conjectures, the more I look for patterns and think like a mathematician. Wanting to give my students the opportunity to experience this type of mathematical thinking, I developed the unit to allow students to make the conjectures for almost all theorems and test their own conjectures.

According to Cirillo and Herbst (2012) allowing students to make their own conjectures can be as easy as presenting students with a diagram of the given information in a theorem and allowing the students to determine what the conclusion or proven statement could be. In several of the lessons, I incorporated this practice to allow students to think like mathematicians instead of simply regurgitating information or imitating a process. Developing the lesson for the equation of a circle on the coordinate plane was very accessible using the different techniques I learned from the work of Cirillo and Herbst (2012) as well as the different discussions and examples we were exposed to in this class.

Another goal with creating this unit is to allow students to make the connections between the properties of circles, and in doing so, create ownership of the material. I feel this goes hand in hand with making conjectures and testing them. Using different technologies makes the testing of conjectures much more efficient as it cuts the time down significantly. Students will be using GeoGebra, Geometer's Sketchpad, and TI-84 Silver Plus Edition Calculators throughout the unit.

While developing this unit, I continually reviewed the Common Core State Standards document. Instead of trying to incorporate too much curriculum in too little time, I decided to focus on four to five standards that encompassed the properties of circles. The following standards are addressed in the unit:

- (G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed* *angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- (G-C.3) Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- (G-C.5) Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant proportionality; derive the formula for the area of a sector.
- (G-GPE.1) Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- (G-SRT.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

These same five standards will also be assessed through both summative and formative assessments throughout the unit. The summative assessments consist of a Mid-Unit Assessment and an End of Unit Assessment. There are several types of formative assessments used within the unit including board partners, board work, thumbs up side down, journal writing, ticket out the door, two 5-minute checks, and questioning TEAMs while they are working cooperatively during the lessons. The word TEAMs in my classroom is simply an acronym for "Together Everyone Achieves More." Wanting students to do more than just group work, the expectations of the TEAMs and roles within the TEAMs will be well established before this unit is facilitated to ensure a more collaborative process. For that matter, each of the different types of formative assessments are well established within the classroom as these are things we do from the beginning of the year.

As the instructor my role within the unit will be more of a facilitator and coach. A goal of mine this year has been to answer students' questions with questions to allow the students to answer their own questions. While developing this unit, I kept this in mind and will continue to revisit this goal during implementation of the unit. Students will be TEAM members working collaboratively towards a common understanding as well as be expected to coach one another pushing each other to persevere throughout the unit. Students will be encouraged to try different conjectures and ideas on the different technologies that will be available to them in order to develop a deep understanding of the mathematics.

The unit is designed with seven lessons spread out over ten days, two review days, and two summative assessment days for a total of fourteen 55-minute class periods. This unit traditionally takes fifteen days with little retention. I am excited to see the product this spring after implementation of the unit. The unit overview table on the following page gives the basic unit design followed by the lesson plans, 5-minute checks, summative assessments, and unit reflection.

Overview of Circle Unit

Day of Unit	Lesson	Standard	Homework
1	Properties of Tangents of a Circle day 1	G-C.2, G-C.3	2 problems at the end of the TEAM sheet
2	Properties of Tangents of a Circle day 2	G-C.2, G-C.3	Page 656 (18, 19, 21, 25, 27-30, 34)
3	Finding Arc Measure	G-C.2	Page 661 (1-10, 11-13, 16-19, 24)
4	Properties of Chords and their Applications day 1	G-C.2, G-SRT.5	Journal entry about how to find the center of an ancient plate.
5	Properties of Chords and their Applications day 2	G-C.2, G-SRT.5	Page 667 (1-10, 15, 18-20)
6	**5 minute check #1** Inscribed Angles and Polygons day 1	G-C.2, G-SRT.5	Journal entry about how to find the angle measure of an inscribed angle.
7	Inscribed Angles and Polygons day 2	G-C.2, G-SRT.5	Page 676 (6-9, 11,12, 17-19, 37, 38)
8	Review day with board partners based on 5 minute check from day 6 and journal entry on day 7.		
9	**Mid Unit Summative Assessment**	G-C.2, G-C.3, G-SRT.5	
10	Apply Other Angle Relationships in Circles	G-C.2, G-C.3	Page 683 (1-13, 16-18, 20)
11	Find Segment Lengths in Circles	G-SRT.5, G-C.3	Page 692 (1-11, 17, 18, 20)
12	**5 minute check #2** Write and Graph Equations of Circles	G-GPE.1	Problems 1-4 on TEAM sheet
13	Review day based on 5 minute check from day 11 and 4 problems from TEAM sheet on day 12.		
14	**Summative Assessment over Unit**	G-C.2, G-C.3, G-C.5 G-GPE.1, G-SRT.5	

Geometry—(9-12)—(Circles day 1 and 2)
(Properties of Tangents of a Circle)
See all handouts attached

Objectives:	After completing the lesson, students will be able to identify and describe properties of a tangent to a circle.
Grade Level or Course Name	Geometry 9-12
Estimated Time	2 days
Pre-requisite Knowledge	Pythagorean Theorem, Pythagorean Theorem Converse, Circumscribed and Inscribed Circles, Congruent Triangles
Vocabulary	Circle, Center, Radius, Chord, Diameter, Secant, Tangent, Common External Tangent, Common Internal Tangent, **Theorems:** Line tangent to a circle is perpendicular to the radius of the circle, Tangents from a common external point are congruent.
Materials Needed	TEAM sheet and Ticket Out the Door for each student, Computers with GeoGebra software available for students
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-C.3) Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 5. Use appropriate tools strategically

DAY 1:

<u>Launch</u> (How will you engage students in the content for the day?)
Students will pick up the TEAM sheet when they come in the door. They will be asked to make a conjecture about segments $\overline{AB}$ and $\overline{BC}$ (see TEAM sheet for picture). Students will then share their conjecture with their TEAM member. As a whole group we will list the conjectures on the board and see what ones we think we could prove.

<u>Explore</u> (How will students explore the content for the day?)
Students will explore their conjectures from the launch using GeoGebra software. They will work with their TEAM and work through the rest of the TEAM sheet. As students are working on this, vocabulary will be introduced as needed. As the vocabulary is introduced, students will write the definitions on their TEAM sheet so

This lesson was creating using problems from the Exeter website and the Geometry book by Larson. Problems were modified to be more problem solving in nature.

that they make sense to them. They will also draw an example of each on the given circle.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
As a whole class we will discuss if students found the conjecture to be true or not. We will list the two theorems on the TEAM sheet. Students will fill out the ticket out the door. This will be used as a formative assessment.

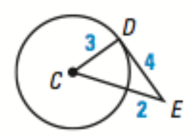
<u>Extension(s)</u>
The homework problems on the TEAM sheet will be given to students as homework. In the homework, students are asked to apply one of the two theorems we found today. The other theorem will be investigated more tomorrow.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
Questioning and checking in with TEAMs as well as the ticket out the door will be used as formative assessment.

<u>Strategies to support English learners</u>
Diagrams will be used. The TEAM sheet allows English learners the opportunity to see what is being asked. Students will be allowed to use the Multilanguage Mathematics Dictionary.

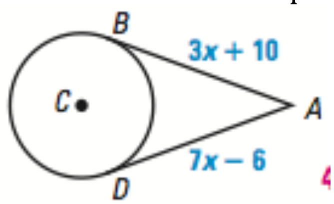
DAY 2:

<u>Launch</u> (How will you engage students in the content for the day?)
Pose the question: Do you think $\overline{DE}$ is tangent to circle C? Why or why not? How could you prove or disprove your conjecture?



<u>Explore</u> (How will students explore the content for the day?)
Students will work with board partners and peer coaching. Students will be asked to find the variable in each problem.

1.



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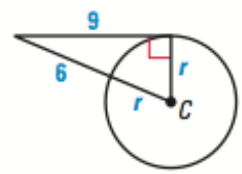
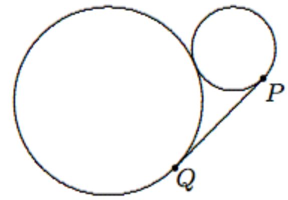


Once students are done with the board partner activity, they will work at their seats with their board partner on the following problem. They will be allowed to talk with other pairs as they work through the problem. (This problem was obtained from the Phillips Exeter Academy Curriculum.)

1. A circle with a 4-inch radius is centered at A, and a circle with a 9-inch radius is centered at B, where A and B are 13 inches apart. There is a segment that is tangent to the small circle at P and to the large circle at Q. It is a common external tangent of the two circles. What kind of quadrilateral is $PABQ$? What are the lengths of its sides?



<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
I will choose a few pairs to go to the board and write how they solved the problem. As a class we will look for similarities and differences. I will choose who goes to the board based my observations during and questioning as students are working on the problem.

<u>Extension(s)</u>
How could you prove the tangent to a circle theorem?

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
Questioning and checking in with pairs during both the board partner activity and the second activity will be used as formative assessment.

<u>Strategies to support English learners</u>
Diagrams will be used and statements written on the board as well as stated verbally. Students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Homework will involve students extending what we did over the past two days and apply these concepts to other problems. Homework is page 656 (18, 19, 21, 25, 27-30, 34)

Key Ideas

Key ideas/important points	Teacher strategies/actions
Making two conjectures that lead to the two theorems about tangents.	Teacher will facilitate the TEAM activity and ask questions that lead the students in the correct direction for their conjectures and "showing" how they are true.
Two radii of two different circles whose endpoints are on a common external tangent create a trapezoid.	Teacher will use questioning to lead students to the idea of a trapezoid.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
What is the longest chord in any circle?	The Radius. Diameter.	What do we know about the location of the endpoints of a chord? Does radius fit this?
How many external tangents can a pair of circles have?	Infinite or one	Can you show me with a picture how this is true?
If the radius is perpendicular to the tangent line at the point of tangency, what can you say about the relationship between a diameter and the tangent line?	That the diameter must also be perpendicular to the tangent line.	What would be true about two different tangent lines whose points of tangency are at each end of a diameter?
If a radius is perpendicular to a tangent, what kind of triangle can we make? What theorem would help us determine lengths of different segments in that shape?	Right Triangle Pythagorean Theorem	How are the lengths of the radius and the tangent segment related to the distance from the external point to the center of the circle?
What happens if you connect the two centers with a segment?	It makes a quadrilateral.	What specific quadrilateral does it make?
How would the two radii be related to the common external tangent? What conclusion can you make about the two radii based on this?	They are perpendicular. The two radii are then parallel.	How could you use this information to find the length of the common external tangent segment?

Misconceptions, Errors, Trouble Spots

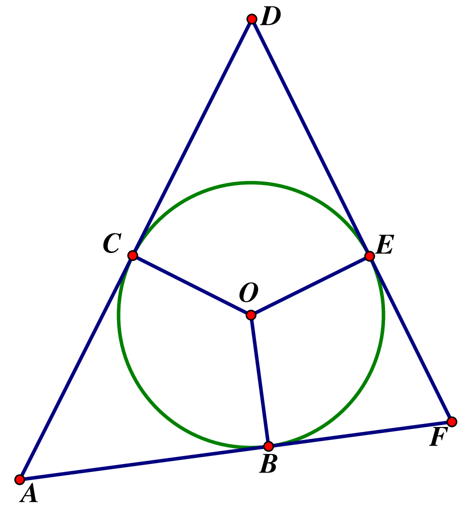
Possible errors or trouble spots	Teacher question/actions to resolve them
Students will try to use the IT theorem instead of the Pythagorean Theorem.	Can you prove that the two segments are equal? What two segments could be equal?
Students will think there are infinite or only 1 internal common tangent lines.	Ask students to verify their conclusion with a diagram.
Students may think that the shape made in day 2 activity #2 is a parallelogram.	What has to be true about the angles of a quadrilateral in order to be a parallelogram?
Students may not realize that this type of trapezoid can be cut into a rectangle and a right triangle.	This concept will be prepped when we work with different types of quadrilaterals on the unit before this one. I will refer to our findings from that unit.

This lesson was creating using problems from the Exeter website and the Geometry book by Larson. Problems were modified to be more problem solving in nature.

Name ________________________
Date _____________ Period ____

Properties of Tangents TEAM sheet

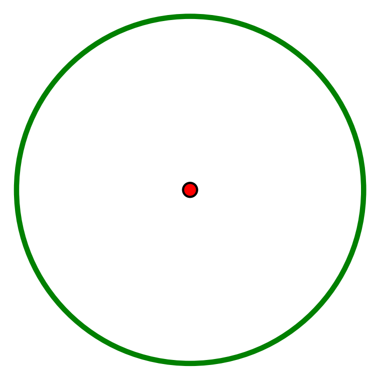
1. Given ΔADF with inscribed circle O, make a conjecture about the relationship between $\overline{AB}$ and $\overline{AC}$.



2. With your assigned TEAM member, please get a computer and create your own ΔADF with inscribed circle O using GeoGebra. Test your conjecture. Does it hold true? Explain why you think this is so.

Vocabulary:

Circle:



Center:

Radius:

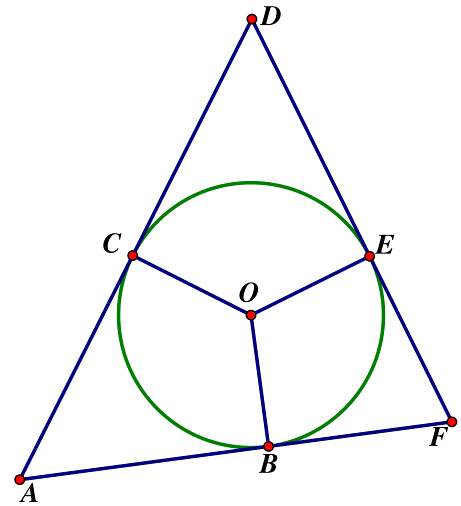
Chord:

Diameter:

Secant:

Tangent:

3. Given ΔADF with inscribed circle O and that $\overline{OC}$ is a radius, make a conjecture about the relationship between $\overline{AD}$ and $\overline{OC}$.



4. Use GeoGebra to explore this conjecture. Test your conjecture. Does it hold true? Explain why you think this is so. (In your explanation use the vocabulary we discussed when appropriate.)

Theorems:
Tangent-Radius:

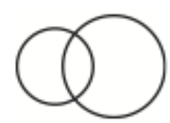
Common External Point Tangents:

Homework:
1. Common Tangents can be a line, ray, or segment that is tangent to two coplanar circles. Given the three pairs of circles below, tell how many common tangents each pair has. Draw them.

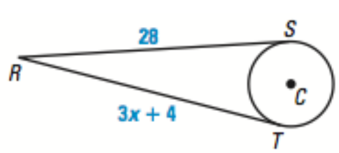
a.

b.

c.



2. $\overline{RS}$ is tangent to circle C at S and $\overline{RT}$ is tangent to circle C at T. Find the value of x.



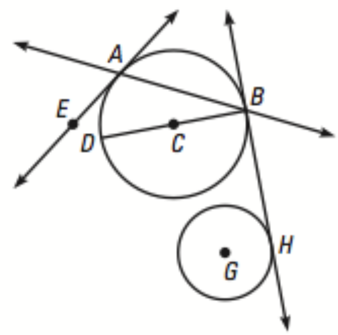
Name ____________________
Date ___________ Period ___

Properties of Circles Day 1 Ticket Out the Door

Given the diagram, name an example of each of the following:

1. A center of a circle. ________
2. A radius of a circle. ________
3. A chord of a circle that is not a diameter.

   ____
4. A diameter of a circle. _______
5. A secant of a circle. _______
6. A tangent of a circle. _______
7. A point of tangency. _______
8. A common tangent. _______



Name ____________________
Date ___________ Period ___

Properties of Circles Day 1 Ticket Out the Door

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   ____
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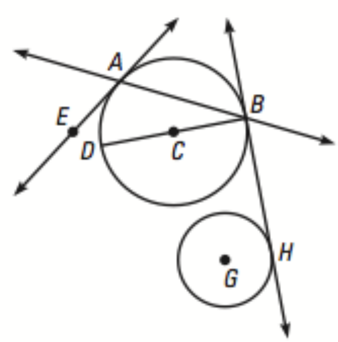
6. A tangent of a circle. _______
7. A point of tangency. _______
8. A common tangent. _______

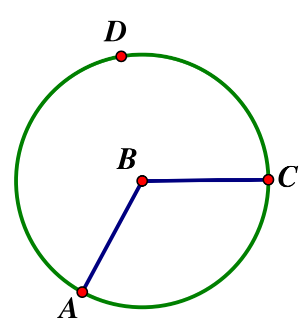


Geometry—(9-12)—(Circles Day 3)
(Finding Arc Measure)

Objectives:	At the end of this lesson, students will communicate by naming arcs and by relating arc measure with central angle.
Grade Level or Course Name	Geometry 9-12
Estimated Time	1 day
Pre-requisite Knowledge	Definitions of: Circle, Radius, Diameter, Angle, Degree, Center of Circle, Congruence
Vocabulary	Central Angle, Minor Arc, Major Arc, Semicircle, Measure, Congruent Circles, Congruent Arcs
Materials Needed	Geometer's Sketchpad for whole class with projector
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 4. Model with mathematics, 5. Use appropriate tools strategically, 6. Attend to precision

<u>Launch</u> (How will you engage students in the content for the day?)

Pose the question: Think of your favorite type of pizza. You and I are going to split that type of pizza. Looking at the diagram that has been cut into two pieces, which piece would you rather have?



I will then give them the smaller piece. They will say no I wanted the other one. We will then discuss the difference in the two pieces. How do we name these two different pieces so that we both know which piece we are talking about?

I will extend this into semicircles.

<u>Explore</u> (How will students explore the content for the day?)

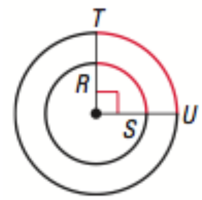
We will define in our notes the vocabulary that came up in our launch. These definitions include: Minor arc, major arc, semicircle, and central angle.

I will explain to students that we measure the arcs in degrees and that we say a full arc, or the total measure of the circumference of a circle in degrees is 360 degrees. I will ask them to think about how we might determine the degrees of a part of the circle. They will write this down and share with their row partner. We will then discuss this as a class.

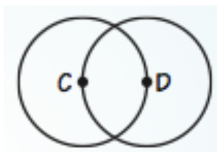
I will then ask students to think about how we could define congruent arcs. Again, they will write this down and compare with their row partner. We will discuss as a whole group.

With their partner, students will answer the two questions below.

1. In the diagram, determine the measure of $\overset{\frown}{RS}$ and $\overset{\frown}{TU}$. Are the two arcs congruent? Explain your conclusion.



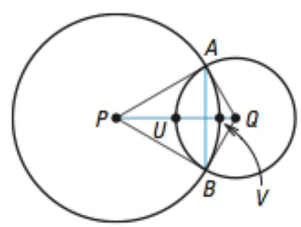
2. Determine if the two circles are congruent. Explain your reasoning.



<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
Students will switch partners and compare their answer to the two questions. As a whole class we will make a final conclusion on both.
Students will be asked to list something you learned, something they want to investigate further, and a question you have for their ticket out the door.

<u>Extension(s)</u>

CHALLENGE In the diagram shown, $\overline{PQ} \perp \overline{AB}$, $\overline{QA}$ is tangent to $\odot P$, and $m\overset{\frown}{AVB} = 60°$. What is $m\overset{\frown}{AUB}$? **120°**



<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
While students are working on the different problems, I be checking in with them and asking questions. We will use thumbs up/side/down. I will review the ticket out the door and the work they did on the problems during the class.

<u>Strategies to support English learners</u>
Due to the amount of vocabulary, I will work with the ELL instructor and give the vocab to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Homework will be page 661 (1-10, 11-13, 16-19, 24)

Key Ideas

Key ideas/important points	Teacher strategies/actions
The importance of common language	During the launch show students how confusing it is if we don't speak the same language.
Congruent circles	Pose the questions written in the explore section and ask questions as student need the guidance.
Finding measures of arcs on a circle	Have students conjecture about how we could assign degrees based on the fact the that whole circle has 360 degrees.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
What happens when we don't have a common language.	We can't communicate or the communication is confusing.	This is why it is important to name geometric shapes in the same way.
How many degrees surround a point?	360 degrees	If there is also 360 degrees in a circle, how could we relate the degrees of the circle to the angle made at the center of the circle related to the arc?
Why do we need to use three points on the circle for a major arc and a semicircle?	Because we need to know if we are talking about the bigger arc or the smaller arc. We also need to know which semicircle we are talking about.	Can we use three points for a minor arc? Why is it not necessary to do this?
What is the definition of congruence? What part of the definition of congruence do all circles satisfy?	Same shape same size. All circles satisfy the same shape.	How could we then define congruent circles? What would it take for two circles to be the same size? Is there more than one way to sow circles are congruent?

This lesson was creating using problems from the Exeter website and the Geometry book by Larson. Problems were modified to be more problem solving in nature.

Misconceptions, Errors, Trouble Spots

Possible errors or trouble spots	**Teacher question/actions to resolve them**
Students will think that if arcs have the same degree measure that they are congruent.	If you have a large pizza and a small pizza and take ¼ of each, which has more crust? What does this do to your idea about congruent arcs?
Students may think that you can name an arc by its central angle.	Have another student who is labeling correctly to explain why they are labeling the way they are.

This lesson was creating using problems from the Exeter website and the Geometry book by Larson. Problems were modified to be more problem solving in nature.

Geometry—(9-12)—(Circles day 4 and 5)
(Properties of Chords)
See all handouts attached

Objectives:	Students will discover and apply properties of chords.
Grade Level or Course Name	Geometry 9-12
Estimated Time	2 days
Pre-requisite Knowledge	Chord, arc, semicircle, congruent circles, congruent arcs, perpendicular bisector
Vocabulary	Corresponding chords, **Theorems: 1.** In the same circle, or in congruent circles, two minor arcs are congruent iff their corresponding chords are congruent. 2. If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. 3. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc. 4. In the same circle, or in congruent circles, two chords are congruent iff they are equidistant from the center.
Materials Needed	TEAM sheet for each student and Geometer's Sketchpad with projector for class use.
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-SRT.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 4. Model with mathematics, 6. Attend to precision, 7. Look for and make use of structure, 8. Look for and express regularity in repeated reasoning

Day 1:
<u>Launch</u> (How will you engage students in the content for the day?)
Students will pick up the TEAM sheet and make a conjecture about the first statement.

<u>Explore</u> (How will students explore the content for the day?)
Students will draw three points on a piece of paper and exchange it with another student. They will have rulers, compasses, and protractors available to attempt to make a circle. In their journals they will record what happened during this attempt.

Using Geometer's Sketchpad on the projector, we will explore the four theorems:

Theorems:
1. In the same circle, or in congruent circles, two minor arcs are congruent iff their corresponding chords are congruent.
2. If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.
3. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
4. In the same circle, or in congruent circles, two chords are congruent iff they are equidistant from the center.

Students will read the Sprinkler Problem. Individually students will write their thoughts and how the four theorems we discovered could aid in determining where the sprinkler should be placed.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
Students will share out what they wrote in their journals about the sprinkler problem. Students will be asked to attempt the sprinkler problem before they come to class tomorrow and to write down all the different ways they attempt it even if they were not successful attempts.

<u>Extension(s)</u>
How could archeologists find the size of a plate that was found in ancient ruins when over half of the plate is missing?

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
During the lesson I will be asking students questions as they work to construct a circle with the points their partner gave them. At the end of the lesson I will ask them to describe to a student who was absent that if you have two chords in a circle how can you tell which one is closer to the center of the circle. This will be turned in before they leave.

<u>Strategies to support English learners</u>
Due to the amount of vocabulary, I will work with the ELL instructor and give the vocab to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. The TEAM sheet will be used as well as collaborative learning. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Day 2:
<u>Launch</u> (How will you engage students in the content for the day?)
Students will be asked to take out their journals and the Sprinkler problem. They will be asked to see if there is anything they can add after reviewing what they had already tried.

<u>Explore</u> (How will students explore the content for the day?)
Students will be placed into TEAMs of 3 and will be exploring the sprinkler problem. As they work through it I will be asking questions that will scaffold the problem as well as check for understanding.

As a whole class we will discuss different methods for solving the Sprinkler Problem. Students will then be asked to do the Extension Problem on the back of the TEAM sheet.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
Students will rephrase the four theorems taking turns with a partner. As a whole group we will discuss how these theorems helped us with the Sprinkler Problem and the Extension.

<u>Extension(s)</u>
See TEAM sheet.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
Thumbs up/side/down will be used periodically. As I facilitate the lesson I will be asking questions that check for understanding. Students will share out how they were able to complete both problems.

<u>Strategies to support English learners</u>
Due to the amount of vocabulary, I will work with the ELL instructor and give the vocab to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. The TEAM sheet will be used as well as collaborative learning. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Homework: page 667 (1-10, 15, 18-20)

This lesson was creating using problems from the Geometry book by Larson and the website http://geometry.mrmeyer.com*. These problems were modified to make them more problem solving in nature.*

Key Ideas

Key ideas/important points	Teacher strategies/actions
Congruent Chords	Teacher will use Sketchpad and questioning to make conjectures and test them about congruent chords.
Diameter bisects chords if it is perpendicular to them	Teacher will use Sketchpad and questioning to make conjectures and test them about congruent chords.
Through three points we can draw a circle.	Teacher will facilitate while students try to draw a circle with the 3 points given to them by their partner.
Chords that are the same length are the same distance from the center of the circle.	Teacher will use Sketchpad and questioning to make conjectures and test them about congruent chords.
In the same circle with two chords that have different lengths, the longer chord is closer to the center of the circle.	Teacher will use Sketchpad and questioning to make conjectures and test them about congruent chords.
Finding the center of a circle given only three points.	Teacher will facilitate while students work through the Sprinkler problem and its extension.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
If you pick any three desks in this room, is it possible for me to stand so that I am exactly the same distance from each desk?	No or yes with various explanations.	If no, we would set it up and see if we could find a spot using string. If yes, I would then ask, what would be true about where I am standing? What part of a circle would I represent?
What is true about congruent arcs? How do you think this relates to the corresponding chord of those arcs?	Congruent arcs have the same measure and are in the same or congruent circles making them the same length. Looking at the picture, it seems that chords that correspond to the same arcs are congruent.	How could you prove this idea? Would You need more information?

What is the longest chord in a circle? How close is it to the center? Where do you think the shortest chord would be located?	Radius or Diameter It is passes through the center of the circle. The chords seem to get shorter the further away it is from the center. I would say the shortest chord is the chord that is the furthest away from the center.	If radius, I would reteach by asking the student to define a chord and see if radius meets its requirement. If diameter, I would encourage them to find a really short chord and see if their conjecture appears to be true.
How do we measure the distance between a point and a line?	From the point we drop a line perpendicular to the line. The distance from the point to the point of intersection is the distance the point is form the line.	If two chords have the same distance from the center, what do you think is true about the two chords?
A diameter cuts a circle into two congruent halves. If that diameter intersects a chord, what do you think needs to be true in order for the diameter to bisect the chord?	They will probably guess some different things like that it has to be in the middle of the chord and so a diameter could be perpendicular to bisect it.	I would encourage them to draw the central angle that goes with the chord. What kind of triangle does this make? (Isos) What do we know about the altitude on an Isos triangle? In your picture what property of the circle is this altitude? How is it related to the diameter?

<u>Misconceptions, Errors, Trouble Spots</u>

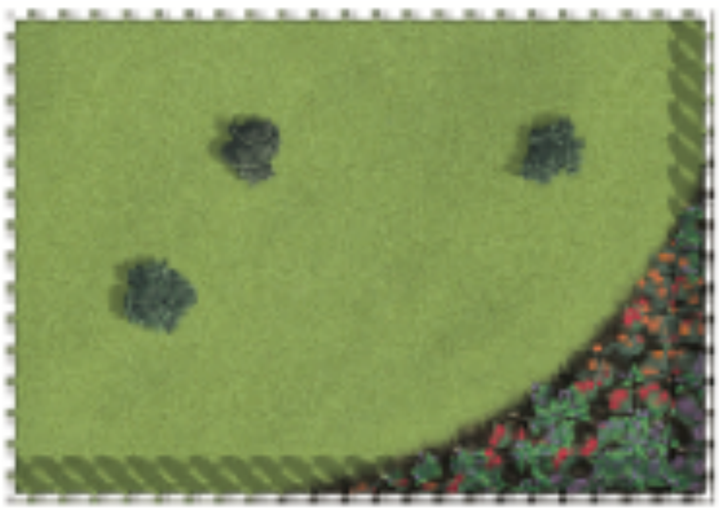
Possible errors or trouble spots	**Teacher question/actions to resolve them**
Students could use one of the three points as the center not understanding that they are on the circle not in…	Reteach that the three points will be on the circumference of the circle. Could give the example that if there are 3 students in the class at three different desks, where should I stand so that I am the same distance from all 3 of them.
Students will try to use the ruler and spend a lot of time trying to find the distance from each point instead of using the idea of perpendicular bisector would be a diameter.	Teacher asks the questions: What postulate guarantees that we have a point? So if two lines intersect at a point, on a circle, what always passes through the center? How many are there? So if we need to find the center, how many diameters would we need? Thinking about the theorems we have, how could we be certain we have a diameter?
For some students, more scaffolding will be needed.	Let students us manipulatives, set up the scenario with pennies and let them see what they can do. Let them use GeoGebra to explore.

Name ________________________
Date ______________ Period____

Properties of Chords TEAM Sheet

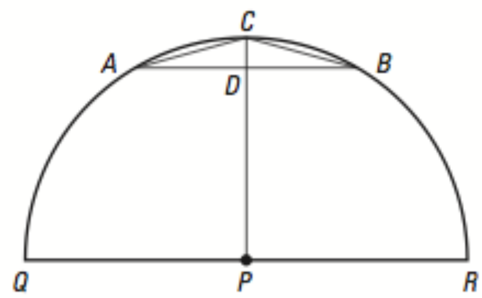
A student made the claim that a circle can be drawn through any three points. Do you agree or disagree with this? Why?

Three bushes are arranged in a garden as shown in the picture. Where should you place a sprinkler so that it is the same distance from each bush? Explain how you arrived at this answer so that someone who was absent would understand.



Extension:

REASONING In the diagram of semicircle $\widehat{QCR}$, $\overline{PC} \cong \overline{AB}$ and $m\widehat{AC} = 30°$. *Explain* how you can conclude that $\triangle ADC \cong \triangle BDC$.



Geometry—(9-12)—(Circles day 6 and day 7)
(Inscribed Angles and Polygons)
See all handouts attached

Objectives:	After completing this activity, students will be able to use a central angle to find the measure of the inscribed angle that intercepts the same arc. Using this concept they will discover relationships about quadrilaterals inscribed in a circle as well as angles inscribed in a semicircle.
Grade Level or Course Name	9-12 Geometry
Estimated Time	2 days
Pre-requisite Knowledge	Central Angle, Secants to a circle, chords, arc measure, semicircle
Vocabulary	Inscribed Angle, Intercepted Arc, Inscribed Angle, Inscribed Polygon, Circumscribed Circle **Theorems**: Inscribed Angle Theorem, Inscribed Angles Intercepting the same arc Theorem, Inscribed Angle in a Semicircle Theorem, Opposite Angles in an Inscribed Quadrilateral Theorem
Materials Needed	TEAM sheet and Calculator Activity Sheet, one of each per student, class set of computers with GeoGebra available for students to use, compasses, Classroom set of TI-84 Silver Plus Edition Calculators.
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-SRT.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 4. Model with mathematics, 5. Use appropriate tools strategically, 7. Look for and make use of structure, 8. Look for and express regularity in repeated reasoning

Day 1:

Launch (How will you engage students in the content for the day?)

Ask students what they believe is true about an inscribed angle and its intercepted arc. Have them make a conjecture and compare with a row partner.

This lesson was adapted and modified from the Texas Instrument Website.

<u>Explore</u> (How will students explore the content for the day?)
Students will be working with CabriJr on TI-84 Silver Plus calculators. They will test their conjecture made at the beginning of the period.

> **Inscribed Angle Theorem**: The measure of an inscribed angle is half the measure of its intercepted arc.
>
> **Inscribed Angles Intercepting the same arc Theorem**: If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
Students will be asked to answer the 8 questions on the second page of the calculator sheet. Selected students will be asked to go to the board and Think Aloud for each problem.

<u>Extension(s)</u>
Prove the Inscribed Angle Theorem.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
The sheets will be collected for their calculator activity. These will be used as a formative assessment for the next day's activity. (This is a day where not much formative assessment will happen in class as students will have many questions about how to use the calculators so much of my time will be spent answering those. If our full-time sub is available that day, she will come in and help. I prep her on how to use the calculators so she can help answer questions.)

<u>Strategies to support English learners</u>
I will work with the ELL instructor and give the two activities to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. The TEAM sheet will be used as well as collaborative learning. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Journal entry about how to find the angle measure of an inscribed angle.

This lesson was adapted and modified from the Texas Instrument Website.

Day 2:

<u>Launch</u> (How will you engage students in the content for the day?)

(There is a TEAM sheet to accompany this. It is question number 1.) Using GeoGebra or a compass, construct a circle. Create a diameter AB and another diameter CD. Connect the endpoints to create a quadrilateral $ACBD$. What type of quadrilateral is this? How do you know? Could you prove this? Explain in complete sentences.

<u>Explore</u> (How will students explore the content for the day?)

Students will use a compass and/or GeoGebra to discover the Angle Inscribed in a Semicircle Theorem and Opposite Angles in an Inscribed Quadrilateral Theorem.

> **Inscribed Angle in a Semicircle Theorem**: If a right triangle is inscribed in a circle, then the hypotenuse is a diameter. An angle inscribed in a semicircle is a right angle.
>
> **Opposite Angles in an Inscribed Quadrilateral Theorem:** A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)

As a whole class we will share out what each TEAM discovered while working through the TEAM activity. We will then formally write the two theorems out. Students will then work at the board with a board Partner to practice problems. Use problems 3, 10, 13, 14 on page 676.

<u>Extension(s)</u>

Write a plan for a proof for the Opposite Angles in an Inscribed Quadrilateral Theorem.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)

I will ask questions of students as I facilitate to check for understanding. At the end of class, students will work examples at the board with peer coaching. This will give me an opportunity to check in quickly with each student to check for understanding.

<u>Strategies to support English learners</u>

I will work with the ELL instructor and give the two activities to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. The TEAM sheet will be used as well as collaborative learning. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: page 676 (6-9, 11,12, 17-19, 37, 38)

This lesson was adapted and modified from the Texas Instrument Website.

Key Ideas

Key ideas/important points	Teacher strategies/actions
Measure of an Inscribed Angle Theorem	I will demonstrate/facilitate the use of CabriJr on the calculators. Once students set up this theorem, I will encourage them to try the next on their own.
Inscribed Angles Intercepting the same arc Theorem	I will facilitate the second part of the calculator activity checking for understanding and use of the calculators to enhance the meaning of the theorem.
Inscribed Angle in a Semicircle Theorem	I will facilitate the use of GeoGebra and/or compass on questions 1-2 on the TEAM sheet to discover this theorem asking prompting questions and for students to confirm their conjecture(s).
Opposite Angles in an Inscribed Quadrilateral Theorem	I will facilitate the questions 3-5 on the TEAM sheet to discover this theorem asking prompting questions and for students to confirm their conjecture(s).

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
When the arc measure is fixed what do you notice happens when you move the vertex of the angle around the circle?	The angle measure stays the same no matter where it is.	What happens if you move the vertex somewhere on the original intercepted arc? What happens to the angle measure? How is this related to the angle measure you found the first time?
How many angles can intercept one arc? What is true about all these angles?	One to infinity	Have students draw an example to explain. In ones that have the misconception that there is a fixed number, ask them to keep drawing more until they notice that it can go on forever.
How can you construct a rectangle inscribed in a circle? Is it possible to construct a rectangle inscribed in a semicircle? Why or why not?	Using two diameters and connecting their endpoints will produce a rectangle. No because to have one right angle inscribed in a circle automatically gets a right triangle which is only 180° and a rectangle has 360°.	Ask them to refer to the TEAM sheet and question them on how they obtained their conclusions. Guide them to thinking about these concepts and their connections.

Misconceptions, Errors, Trouble Spots

Possible errors or trouble spots	Teacher question/actions to resolve them
There will be lots of questions on the calculator use.	Have the TI cheat sheet available for students to us. Ask for other adults who are available those hours and have knowledge of the calculators to come and help facilitate. Have patience and encourage students to do the same!
Students may not realize that the inscribed quadrilateral in problem 1 is a rectangle.	Encourage students to use the software and measure the side lengths as well as angles. Have them manipulate the endpoints of the diameters and see that the angles never change. The sides change but opposite sides always remain the same length.
Students may not realize that the opposite sides of an inscribed quadrilateral are supplementary. They may conjecture that they are congruent.	Use software to show an obvious shape where the opposite angles cannot be congruent. Point out number 4 on the TEAM worksheet.

This lesson was adapted and modified from the Texas Instrument Website.

Name ________________________
Date _____________ Period _______

Cabri Jr. Inscribed and Central Angles

1. Draw a circle in the center of the screen. Label the center O.
   - Use the point tool to make three points on the circle. Do not use the radius point of the circle as one of the points. Label the points A, B, and C as in Figure 1a.
   - Construct an angle ($\angle BAC$) using the Segment tool. $\angle BAC$ is an inscribed angle subtended by the minor arc $\overset{\frown}{BC}$. The vertex, A, is on the circumference of the circle and the endpoints BC are the endpoints of $\overset{\frown}{BC}$.
   - Construct an angle ($\angle BOC$) using the Segment tool. $\angle BOC$ is a central angle subtended by the minor arc $\overset{\frown}{BC}$. The vertex, O, is at the center of the circle and the endpoints BC are the endpoints of $\overset{\frown}{BC}$. See Figure 1b.

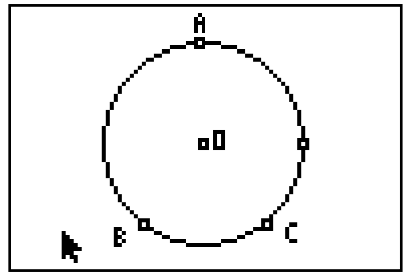


Figure 1a

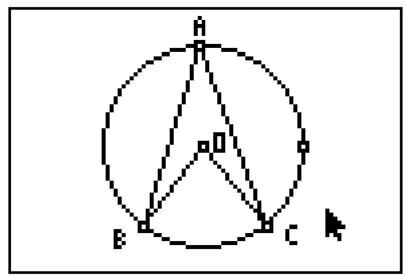


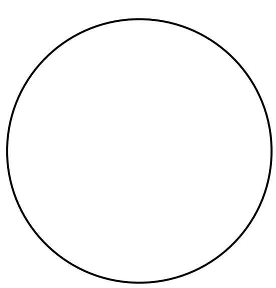
Figure 1b

2. Measure the two angles $\angle BAC$ and $\angle BOC$ using the Angle tool. Record the measurements in the table.
   - Drag point B along the circle.
   - Record the new angle measurements in the table.
   - Continue to drag point B along the circle and record the measurements of the new angles formed.

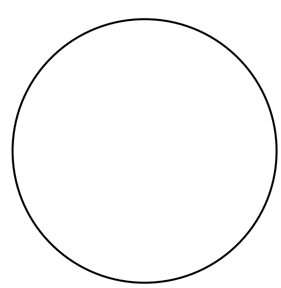
$\angle BAC$	$\angle BOC$

3. Make a conjecture about the relationship between the measure of an inscribed angle and the central angle subtended by the same arc.

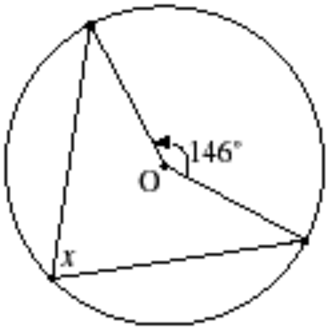
4. Sketch an example of an inscribed angle.

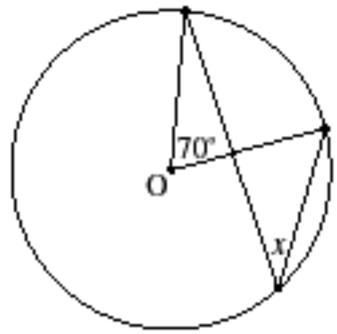


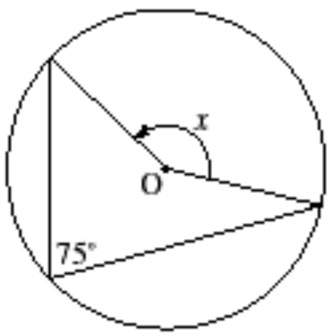
5. Sketch an example of a central angle.

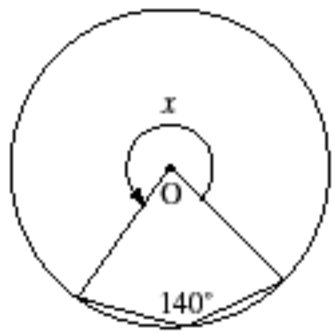


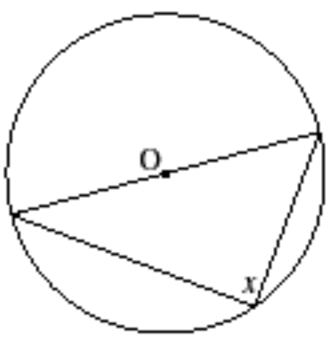
6. Determine each value of *x* or *y*. Point O is the center of each circle.

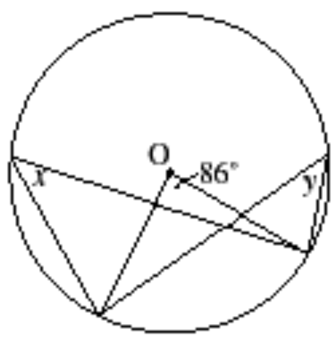


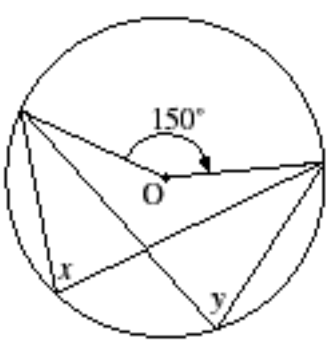


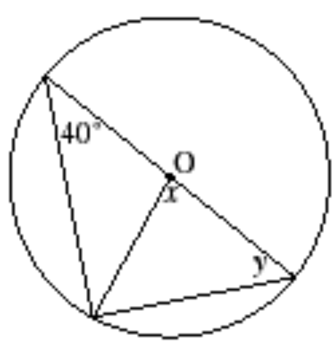










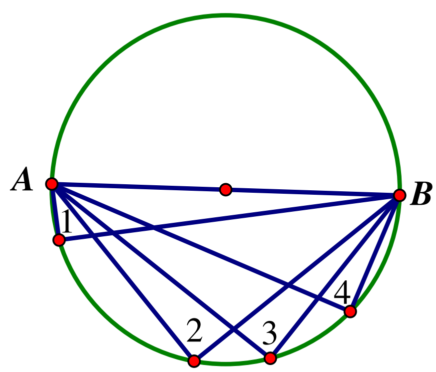


Name ________________________
Date ______________ Period ____

TEAM Activity Sheet Geometry Sec 10.4

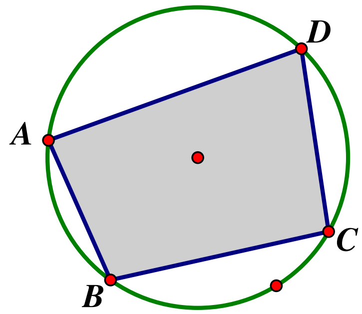
1. Using GeoGebra or a compass, construct a circle. Create a diameter *AB* and another diameter *CD*. Connect the endpoints to create a quadrilateral *ACBD*. What type of quadrilateral is this? How do you know? Could you prove this? Explain in complete sentences.

2. Based on your observation from number 1, what do you think is true about the following angles inscribed in a semicircle? Do you think this works for any circle?



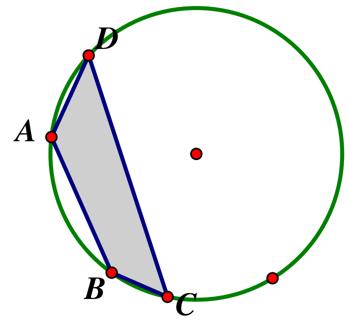
3.

Given: $m\widehat{ABC} = 160°$
$m\angle BAC = 75°$
$m\widehat{ADC} = 200°$
Find: $m\angle D, m\angle C, m\angle B$



4.

Given: $m\widehat{ADC} = 300°$
$m\widehat{BC} = 26°$
$m\widehat{AD} = 30°$
Find: $m\angle A, m\angle B, m\angle C, m\angle D$



5. What do you notice about the relationship between $\angle A$ and $\angle C$ in both 3 and 4? Is the same true for $\angle B$ and $\angle D$? Why do you think this is true?

Geometry—(9-12)—(Circles day 10)
(Other Angle Relationships in Circles)

Objectives:	After completing this lesson, students will understand the relationship between angles and circles.
Grade Level or Course Name	9-12 Geometry
Estimated Time	1 day
Pre-requisite Knowledge	Inscribed angle, arc measure, central angle, diameter, chord, radius, secant line, tangent line
Vocabulary	Tangent Chord Theorem, Angels Inside the Circle Theorem, Angles Outside the Circle Theorem
Materials Needed	A classroom set of computers with GeoGebra available for all students
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-C.3) Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 4. Model with mathematics, 5. Use appropriate tools strategically, 6. Attend to precision, 7. Look for and make use of structure, 8. Look for and express regularity in repeated reasoning

Launch (How will you engage students in the content for the day?)
Pose the question, "We know what happens when we have an angle inside a circle where the vertex is at the center. What do you think is true when two chords intersect so that the vertex is not at the center but is inside the circle?" Using a compass and protractor examine your conjecture.

Explore (How will students explore the content for the day?)
Students will investigate the launch question as well as additional questions that will lead them to discover the three theorems. GeoGebra and Geometer's Sketchpad will be used to allow students opportunities to work with all three theorems.

Theorem one: Launch question.

If two chords intersect inside a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

Theorem two: Draw a circle with a tangent line to it at point P. Ask students how many degrees are in the straight angle of the tangent line. Ask students how many degrees are in the whole circle. What is this relationship? If you draw a chord that is not a diameter with an endpoint at P, what do you think is true about the two angles made with the chord and the tangent? What is their relationship to the arcs they intercept? (Use GeoGebra to explore.)

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

Theorem three: What if we have two secant lines that intersect outside the circle? How many arcs would they intersect? What do you suppose is true about the relationship between these two arcs and the angle made with the two secants? (Use GeoGebra to explore.)

If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.

<u>Summary/Close of the lesson</u> (How will you close your lesson and bring student understanding to a close for the day?)
As a class we will design three column notes that show all the different cases for angle relationship with circles. The columns will be: Type of Angle with definition, Diagram, Theorem with example.

<u>Extension(s)</u>
Prove the Angles Inside the Circle Theorem.

<u>Check for Understanding</u> (How will you assess students throughout and at the end of the lesson?)
I will facilitate the use of GeoGebra and question students for understanding. I will use thumbs up/side/down. The three column notes will tell a lot. I will check their notes and examples.

<u>Strategies to support English learners</u>
I will work with the ELL instructor and give the theorems in 3 column note format to the instructor before the lesson so that they can prep the students for it. Pictures will be used whenever possible. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Page 683 (1-13, 16-18, 20)

Key Ideas

Key ideas/important points	Teacher strategies/actions
Angles Inside the Circle Theorem	Ask students the following question and facilitate their work with GeoGebra: We know what happens when we have an angle inside a circle where the vertex is at the center. What do you think is true when two chords intersect so that the vertex is not at the center but is inside the circle?" Using a compass and protractor examine your conjecture.
Tangent Chord Theorem	Ask students the following question and facilitate their work with GeoGebra: Draw a circle with a tangent line to it at point P. Ask students how many degrees are in the straight angle of the tangent line. Ask students how many degrees are in the whole circle. What is this relationship? If you draw a chord that is not a diameter with an endpoint at P, what do you think is true about the two angles made with the chord and the tangent? What is their relationship to the arcs they intercept? (Use GeoGebra to explore.)
Angles Outside the Circle Theorem	Ask students the following question and facilitate their work with GeoGebra: What if we have two secant lines that intersect outside the circle? How many arcs would they intersect? What do you suppose is true about the relationship between these two arcs and the angle made with the two secants? (Use GeoGebra to explore.)

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
How can you tell if we subtract or sum the two intercepted arcs?	If the angle is inside of the circle we average the two arcs so we sum. If the angle is on the outside of the circle we subtract and divide by two.	What theorems deal only with one arc? When do we know if we need to cut the arc in half or use the actual measure?

What do all three theorems have in common? What are their differences?	They all deal with arc measure and their relationships to angle measures that intercept them.	What theorems deal with two arcs? What theorems divide an arc in half? What theorems use the measure of the arc for the measure of the angle?
How are these three theorems related to the central angle theorem and the Angle inscribed in a Semicircle Theorem?	They all deal with arc measure and their relationships to angle measures that intercept them. The previous ones only dealt with central angles and their relationship to the arc in that it is the same.	How could we prove that an angle inscribed in a semicircle is a right angle using these other theorems?
Does the Angles Inside a Circle Theorem work for Central Angles? Why or why not?	Yes because the two intercepted arcs would be equal. If you find the average of them, it would just equal the measure of one of them.	Can you explain what you mean so that someone who has been gone for the unit would understand?

<u>Misconceptions, Errors, Trouble Spots</u>

Possible errors or trouble spots	Teacher question/actions to resolve them
Students could think that an angle inside the circle that is not a central angle is the same as the measure of the arc.	How do you know which arc to use? Since there are two different arcs, how could we distribute the arc measures in a fair way?
Because all the other types of angles deal with sums, students could think the angles outside of the circle also use the sum of the arcs.	Have students sketch an example on GeoGebra and measure the arcs and angles. Ask students what this relationship is and why it makes sense.
Students may have a difficult time seeing the two intercepted arcs when we have a chord/tangent, a tangent/tangent, and a tangent/secant.	Encourage students to use colored pencils to mark the arcs. Use two different colors and model for them how this can be done.

Geometry—(9-12)—(Circles day 11)
(Finding Segment Lengths in Circles Using Power of Points)
See all handouts attached

Objectives:	Students will: ▪ Articulate the relationship among the three cases that constitute the Power of Points theorem. ▪ Use the Power of Points theorem to solve numerical problems. ▪ Calculate the power of a point.
Grade Level or Course Name	9-12 Geometry
Estimated Time	1 day
Pre-requisite Knowledge	Secant line, Tangent line, Chord, Intercepted Arc measure, Radius, Diameter, Center, Ratios, Proportions
Vocabulary	Segments of a chord, Secant Segment, External Segment **Theorems: Segments of Chords Theorem**: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to product of the lengths of the segments of the other chord. **Segments of Secants Theorem**: If two secant segments share the same endpoint outside the circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. **Segments of Secants and Tangents Theorem**: If a secant segment and a tangent segment share an endpoint outside ac circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.
Materials Needed	Power of Points Lesson Plan from NCTM Illuminations http://illuminations.nctm.org/LessonDetail.aspx?ID=L700 as well as one of each for each student: Computer with Internet connection Chord Problem Overhead Numerical Problems Overhead Numerical Problems Activity Sheet
Iowa Common Core Content Standards	(G-C.2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (G-SRT.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 4. Model with mathematics, 5. Use appropriate tools strategically, 8. Look for and express regularity in repeated reasoning

Launch (How will you engage students in the content for the day?)
See Illuminations Power of Points Soccer Problem.

Explore (How will students explore the content for the day?)
See Illuminations Power of Points Soccer Problem Lesson Plan.
Students will be using the applet for the Soccer Problem and the Power of Points Problem to explore the three theorems

http://illuminations.nctm.org/ActivityDetail.aspx?ID=122

http://illuminations.nctm.org/ActivityDetail.aspx?ID=158

Summary/Close of the lesson (How will you close your lesson and bring student understanding to a close for the day?)
Students will be asked to answer the following as their ticket out the door.
What is the relationship about the three cases? How did the dynamic geometry environment help you to discover the relationship?

As a whole group we will list the three theorems as
(part)(part)=(part)(part)
(sec)(out)=(sec)(out)
(sec)(out)=(tan)(tan)

Extension(s)
See extensions Illuminations Power of Points Soccer Problem Lesson Plan.

Check for Understanding (How will you assess students throughout and at the end of the lesson?)
I will check by questioning students while they work with the applets as well as do whole group questioning periodically. I will use the ticket out the door to assess how much review or clarification is needed on these theorems before we take the test.

Strategies to support English learners
I will work with the ELL instructor and have her help me with the words and directions on the applets that may be troublesome. Pictures will be used whenever possible. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: page 692 (1-11, 17, 18, 20)

This lesson was adapted from the NCTM Illuminations Power of Points Lesson, the Geometry book by Larson, and Foundations of Geometry by Venema.

Key Ideas

Key ideas/important points	Teacher strategies/actions
Segments of Chords Theorem	Facilitate the activity and use of the applets. Ask questions when appropriate to lead student to the fact that the products remain the same. Use transparency for the Chord/Chord portion of lesson and have students Think, Ink, Pair, Share.
Segments of Secants Theorem	Facilitate the activity and use of the applets. Ask questions when appropriate to lead student to the fact that the products remain the same. Give students example problems and have them find a solution.
Segments of Secants and Tangents Theorem	Facilitate the activity and use of the applets. Ask questions when appropriate to lead student to the fact that the products remain the same. Encourage student to think about how all three theorems are alike and different.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
What happens when you move point P outside of the circle? How is this related to when it is inside the circle?	Anything from the segments get longer to the angle gets smaller to the product of the two segments stays the same no matter where the point is.	Encourage them to look at the segments in this case not just angles. How are the two parts of the chords related to the whole secant and the part of the secant outside the circle?
What do all three theorems have in common?	They are all proportional. All deal with a point that creates congruent products.	What happens to the power of point when you move point P so that it is at the center of the circle?
In working with the Soccer Applet, how does the circle help you find the angle with the largest measure?	After what we did with inscribed angles, it helps me find the intersection of the circle and the path, thus creating an inscribed angle.	How does this inscribed angle help you to find the largest angle? Did you first think that it got bigger the closer you got to the goal? When did you notice that it eventually gets smaller while still moving towards the goal?

How could we use the idea of similar triangles to show prove these theorems?	Anything from not sure to students trying it out.	Encourage them to try it. Remind them about the ratios in similarity. Ask them to manipulate the equations we came up with to get ratios instead of cross products.
In working with the Soccer Applet, how does moving the position of the player affect the circle and the angle measure?	The circle gets bigger or smaller depending on which direction the player is moving. The angle measure stays the same.	Would you want to go for the goal when the circle is larger or smaller? Why?

Misconceptions, Errors, Trouble Spots

Possible errors or trouble spots	**Teacher question/actions to resolve them**
Anything using a secant will be troublesome as far as knowing what two segments we are using to find the products.	Have them look at the second applet again and show that all of these theorems are really the same it just depends on where the power of point is. Help them investigate this using the applet so that they can see why we use the outside segment and the whole secant each time.
The use of the applets may cause students to get frustrated.	Encourage them to read the directions that go with the applets and to have patience. Encourage them to try things as they can always reset the applet and may discover something new.
Students may not have enough time to practice problems like I would like them to.	I will have to be flexible and recognize the different needs in the class. Adjustments to the problem and the homework will be made if necessary. An extra half-day may be used to do board partners or dry erase boards if necessary.

This lesson was adapted from the NCTM Illuminations Power of Points Lesson, the Geometry book by Larson, and Foundations of Geometry by Venema.

Geometry—(9-12)—(Circles day 12)
(Equations of a Circle)
See all handouts attached

Objectives:	After completing this activity, students will be able to write the equation for circles on the coordinate gird as well as graph a circle given an equation.
Grade Level or Course Name	9-12 Geometry
Estimated Time	1 day
Pre-requisite Knowledge	Pythagorean theorem, radius, diameter, center of a circle, origin, graphing on the coordinate grid, distance
Vocabulary	Standard equation for a circle
Materials Needed	TEAM sheet for each student, classroom set of computers with GeoGebra available for students.
Iowa Common Core Content Standards	(G-GPE.1) Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them, 2. Reason abstractly and quantitatively, 6. Attend to precision, 7. Look for and make use of structure, 8. Look for and express regularity in repeated reasoning

Launch (How will you engage students in the content for the day?)
Students will pick up their TEAM sheet as they come in the door. They will be asked to think about question 1 which states, "How many points are there on the coordinate plane that are 5 units away from the origin (0,0)? Justify your answer. (You may use graph paper or Geogebra if necessary.)" They will work on this question for 5 minutes on their own.

Explore (How will students explore the content for the day?)
I will ask students how they want to attempt to solve the problem. Students will then be grouped based on what resource, if any, they want to use.

Once students are grouped, they will work through the TEAM sheet answering questions 1-4. As they work, I will facilitate and ask questions of the group to get an idea of understanding.

Summary/Close of the lesson (How will you close your lesson and bring student understanding to a close for the day?)
Based on group observations throughout the activity, at least 2 TEAMs will be asked to share how they solved the problems. Together, we will work through how to communicate these distances with a formula, $(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the

center of a circle and r is the radius. Students will then be asked to come up with their own examples and we will share these working them together.

Extension(s)

The extension questions will be the homework. Students are asked to extend the ideas we came up with in class. They will be allowed to continue to work on them in their TEAM as time permits. The following day they will have the opportunity to compare and discuss what they found on these four problems with their TEAM members.

Check for Understanding (How will you assess students throughout and at the end of the lesson?)

Understanding will be assessed through questioning of TEAMs as I facilitate the activity. Thumbs up/side/down will be used with the equation of a circle. Once we have the equation, I will ask them to make up an example and write it down. They will share this and I will check to see that they understand how to put the center and radius in correctly.

Strategies to support English learners

The ELL instructor and I will get together before hand to frontload vocab and make sure that everything is covered. Pictures will be used whenever possible. The TEAM sheet will be used as well as collaborative learning. Access to technology will be available. Also, students will be allowed to use the Multilanguage Mathematics Dictionary.

Homework: Problems 1-4 on the TEAM sheet.

Key Ideas

Key ideas/important points	Teacher strategies/actions
Realizing that there are an infinite number of points that are 5 units away from the origin.	As students are working, I will ask for justification of the points they find. I will facilitate the activity and encourage the TEAMs as necessary to lead them in the right direction.
How to write the equation for a circle centered at (0,0)	Questioning will be asked to lead students to the correct conclusion. I will encourage them to think of the distance formula and its relationship to the Pythagorean theorem.
How to write the equation for a circle centered at a point that is not at the origin	Similar to the previous idea, I will lead student to the correct conclusion through questioning.

Guiding Questions

Good questions to ask	Possible student responses or actions	Possible teacher responses
Do you think this will work for any point on the coordinate plane? Why?	Yes, because if you use the Pythagorean Theorem you can find the exact distance of 5.	How could you prove this or convince someone who wasn't here that this is correct?
How do the Pythagorean Theorem and its relationship to the distance formula help find the solutions to these problems?	Using the Pythagorean Theorem, I can find points that are five from (0,0). Using the distance formula, I can check that this is true.	How do you know that you have all the points? Do we only get whole number answers when using the Pythagorean Theorem?
How do your ideas support the idea of a circle instead of a square?	Originally, I made a square but now I see that there are points that are not whole number answers.	If I gave you a string that was 5 inches long, how could you use it to find the answer to the original question in number 1?
Are the only answers to this question whole numbers?	No, the Pythagorean Theorem often times gets decimal answers.	Using a string that is 5 units long, how could you use it to answer the original question in number 1?

Misconceptions, Errors, Trouble Spots

Possible errors or trouble spots	Teacher question/actions to resolve them
Assuming that there are only 4 points that work.	Do you think that only whole numbers work for the Pythagorean Theorem or the distance formula? What types of answers do we get on a regular basis?
Thinking that the equation is always $x^2+y^2=r^2$.	Use your formula and see if you get an answer that fits on the circle you have drawn. Or, use GeoGebra to justify your answer. What does it tell you?

Name ______________________________
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Geometry 10.7 TEAM sheet

1. How many points are there on the coordinate plane that are 5 units away from the origin (0,0)? Justify your answer. (You may use graph paper or Geogebra if necessary.)

2. How could you describe to someone how to find the points that are 5 units away from the origin (0,0)?

3. How could you describe to someone how to find the points that are r units away from the origin (0,0)?

4. Does your method work if we want to find all the points that are 5 units away from a different point, say point (3, 2)? How about r units away from point (h, k)?

Homework is on the back of this sheet.

Homework:

1. Find the largest radius of a circle that can be drawn in a right triangle with legs of 6 cm and 8 cm.

2. Find the equation of a circle that passes through the three points (0,0), (0,8), and (6,12).

3. Find an equation for a line that goes through the intersection points of the circles $x^2 + y^2 = 25$ and $(x - 8)^2 + (y - 4)^2 = 65$.

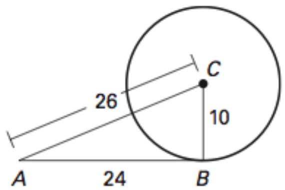
4. The epicenter of an earthquake is the point on the Earth's surface directly above the earthquake's origin. A seismograph can be use to determine the distance to the epicenter of an earthquake. Seismographs tell how far an epicenter is from their location.

   How many seismographs do you think are needed to find the epicenter? How would this information be used to determine where the epicenter is located? Given an example.

Name ______________________________
Date __________________ Period ______

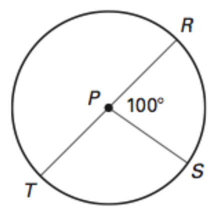
Geometry 5 Minute Check 1: Circles

1. Determine if $\overline{AB}$ is tangent to circle *C.* Explain your reasoning using complete sentences.



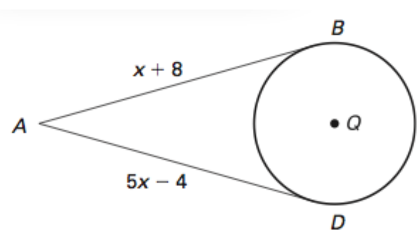
2. Given $\overline{RT}$ is a diameter, find the measure of each indicated arc of circle *P.*

a. $m\overset{\frown}{RS}$ ____________
b. $m\overset{\frown}{ST}$ ____________
c. $m\overset{\frown}{RTS}$ ____________
d. $m\overset{\frown}{RST}$ ____________

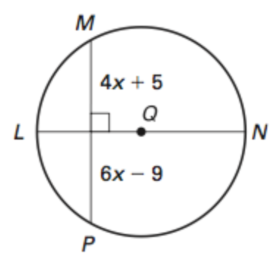


3. Find the value of *x* in each.

a.



b.

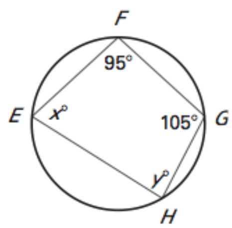


Name ______________________________
Date __________________ Period ______

Geometry 5 Minute Check 2: Circles

1. Find the values of *x, y,* and *z.*

$m\widehat{HEF} = z^\circ$



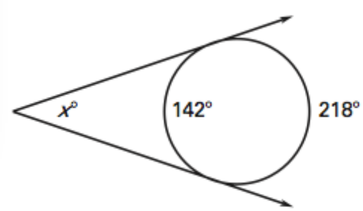
x = __________

y = __________

z = __________

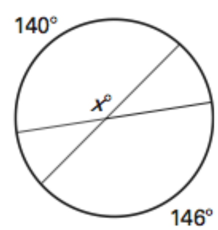
2. Find the value of *x.* Be sure to show all work and how you thought about the problems.

a.



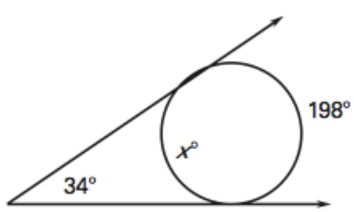
x = __________

b.



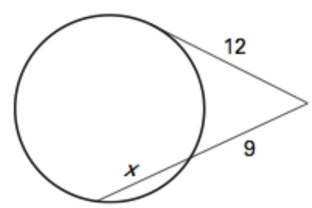
x = __________

c.



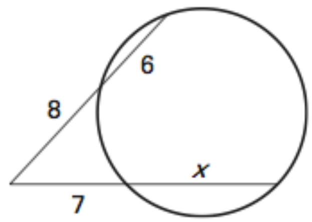
x = __________

d.



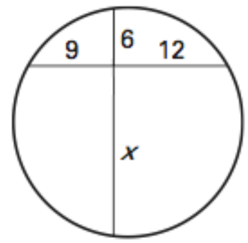
x = __________

e.



$x =$ ____________

f.



$x =$ ____________

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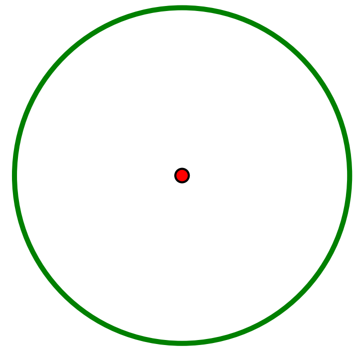
Geometry Mid Unit Assessment: Circles

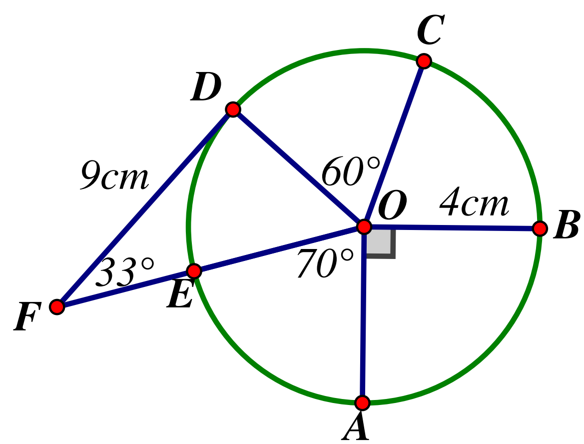
Sketch and label the following.



1. Chord ________
2. Secant ________
3. Tangent ________
4. Diameter ________
5. Radius ________
6. Central Angle ________
7. Minor Arc ________
8. Major Arc ________
9. Semicircle ________

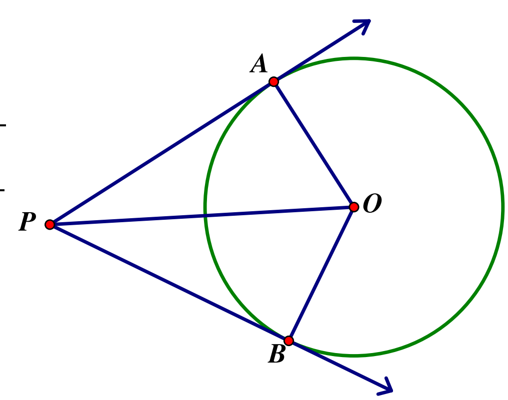
10. Given circle O with radius of 4 cm and tangent segment $\overline{FD}$ whose measure is 9 cm, find the following:

a. $m\widehat{AB}$ = __________
b. $m\widehat{AE}$ = __________
c. $m\angle FDO$ = __________
d. $m\angle DOF$ = __________
e. $m\widehat{DE}$ = __________
f. $m\widehat{CD}$ = __________
g. $m\widehat{BC}$ = __________
h. OF = __________
i. OE = __________



11. $\overline{PA}$ and $\overline{PB}$ are tangents to circle O. Complete the following.

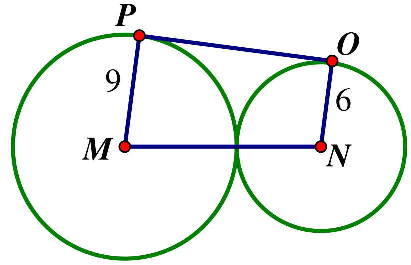
a. $m\angle OAP$ = __________
b. If $m\angle BPO = 36°$, find $m\angle BPA$. __________
c. If $m\angle AOP = 42°$, find $m\angle APB$. __________
d. If $PA = 10\ cm$, find PB. __________



12. Given that $\overline{OP}$ is a common tangent and the circles are tangent to each other. If the radii are 9 and 6, solve for the following.

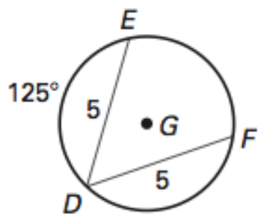
a. MN = ___________

b. PO = ___________

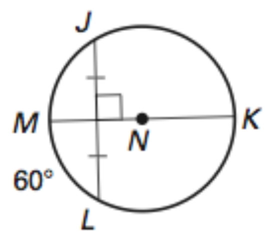


Find the measure of the given arc.

13. $m\widehat{DF}$

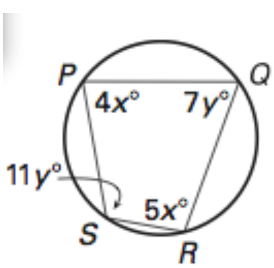


14. $m\widehat{JML}$

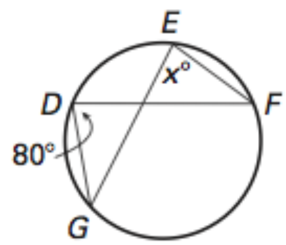


Find the values of the given variables.

15.



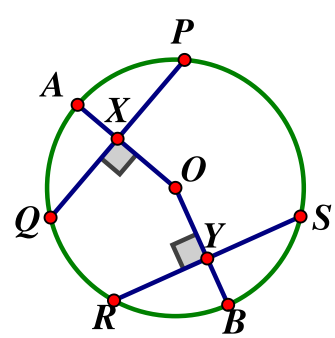
16.



17. Given PQ = 16, OX = 6, OY = 6, and arc QP = 100°, find the following.

a. YS = __________

b. $m\widehat{BS}$ = ___________



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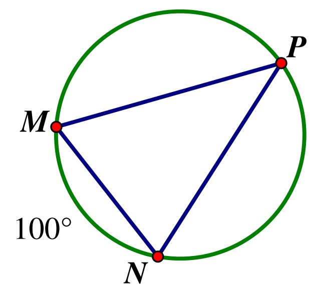
Geometry Unit Assessment: Circles

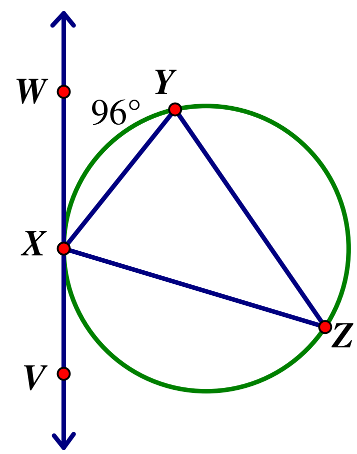
Given ΔMNP is an isosceles triangle with base $\overline{MN}$ and the measure of arc MN is equal to 100°.

1. Name two congruent segments. _________
2. Name two congruent minor arcs. _________
3. What is the measure of $\overset{\frown}{NMP}$. _________
4. What is the measure of $\overset{\frown}{MPN}$. _________



Given: $\overleftrightarrow{WV}$ is a tangent, $XZ \cong YZ$, and $m\overset{\frown}{XY} = 96°$, find:

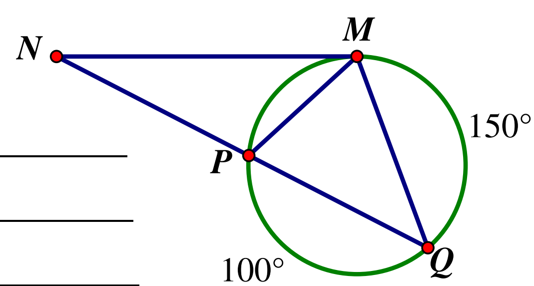
5. $m\angle WXY =$ __________
6. $m\angle XZY =$ __________
7. $m\angle YXZ =$ __________
8. $m\overset{\frown}{YZ} =$ ___________
9. $m\overset{\frown}{XZY} =$ ___________



Given: $\overline{MN}$ is tangent to circle O.
$\overset{\frown}{PQ} = 100°$, $\overset{\frown}{MQ} = 150°$

Find:

10. $\overset{\frown}{PM} =$ ________
11. $m\angle NMP =$ _________
12. $m\angle PQM =$ _________
13. $m\angle NPM =$ _________
14. $m\angle MPQ =$ _________
15. $m\angle MNP =$ _________
16. $m\angle PMQ =$ _________
17. $\overset{\frown}{MPQ} =$ _________



Given: Circle O, $\overline{AC}$ is a diameter
$\overline{AE}$ is a tangent to circle O.
$\overset{\frown}{DC} = 40°$ and $\overset{\frown}{AG} = 70°$

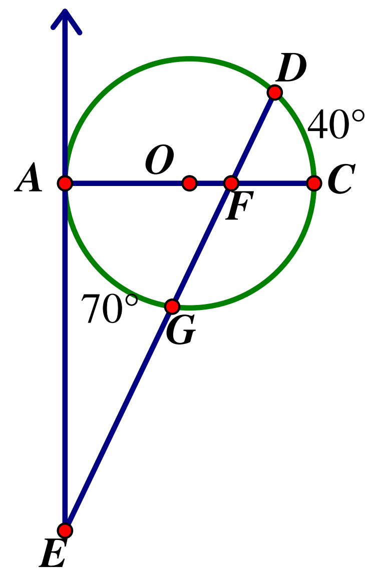
Find:

18. $\overset{\frown}{AD} =$ _______ 19. $m\angle E =$ ________

20. $\overset{\frown}{GC} =$ _______ 21. $\overset{\frown}{ADG} =$ ________

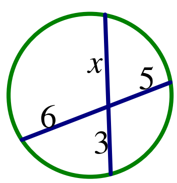
22. $m\angle AFG =$ _______ 23. $m\angle AFD =$ ________

24. $m\angle FAE =$ _______

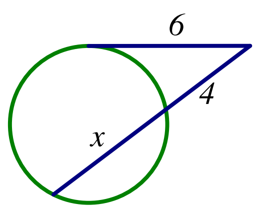


Given the figures below with the chords, tangents, and secants, solve for *x*. SHOW All YOUR WORK!

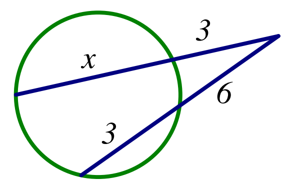
25.



26.

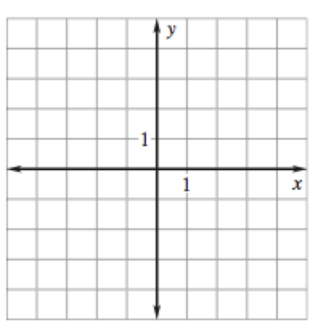


27.

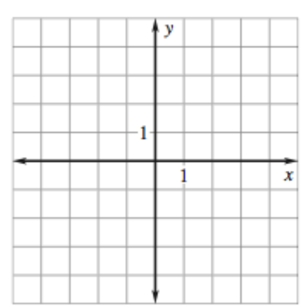


State the center and radius of each circle. Graph each.

28. $x^2 + (y - 1)^2 = 9$



29. $(x - 2)^2 + (y + 1)^2 = 1$



30. A wagon wheel has 14 spokes.

a. What is the measure of the angle between any two spokes?

b. Two spokes in the wagon wheel form a central angle of about 128.5°. How many spokes are between the two spokes?

31. A circle is described by the equation $(x-3)^2 + (y-2)^2 = 25$. Determine whether the line $y = 4x - 13$ is a tangent, secant, secant that contains a diameter, or none of these.

Reflection

In a class conversation this year, it has been said that even though teachers believe in the problem-solving classroom philosophy and theory, they often times do not adopt the actual process within their classrooms. After completion of this unit, I can understand why this might be due to the lack of time needed to make mathematics problematic. I am amazed at how much time was spent researching the different problems let alone creating the lessons and assessments to accommodate those problems.

After compiling several problems, it was necessary to practice the problems myself so that I could modify the lessons to meet the needs of my classroom. Wanting to master the art of questioning forced me to delve even deeper with each problem adding more preparation time. However, I do believe strongly in the problem-solving philosophy and know that the best way to advocate for it is to practice it.

With that being said, I am not completely satisfied with this unit. For the purposes of this class I tried to focus more on problem posing and not as much on assessment and homework. I believe that it is very important to match the assessment with the instruction. Due to the amount of time required to find, modify, and create problems, I was unable to create the perfect assessments. However, my experiences in education help me realize that no unit is perfect and needs to continually be re-evaluated. Even though I have been teaching for many years, this is a new planning process for me. I hope to become more efficient with the process the more I practice.

Currently, as a member of the Response to Intervention (RtI) team for Boone High School, I am researching formative and summative assessments as well as standards based grading. I am excited about the possibility of putting all these ideas together and making this unit on circles richer. I am just getting my feet wet, so to speak, on standards based grading but thought about what I have learned on the subject while creating the assessments. My hope is to develop a deeper understanding on this model so I can adjust my assessments to fit the model. I do not feel like I have enough background yet to implement this but see how it could fit with this unit.

This process was very valuable to my own education. Being a seasoned educator, this was not the first unit I have written. However, it was the first that the main focus was on problem-solving and problem posing. Other units I have written will have problem-solving within the unit but this seems to be more superficial. In creating this unit, I truly tried to be purposeful in the selections of problems in order to create an environment where mathematics is learned through problem-solving. This is very important to me as I want to get away from problem doing and do more problem-solving. When I did not find a problem or problem set that accomplished the desired outcome, I created my own.

I will continue to use the lesson plan template I used for this unit. Working through the lessons and thinking about what problem areas students may have as well as areas that students may need coaching in helped me relate to my students and assess whether the tasks presented produced the desired outcome. This process helped me find problems and make them better to meet the needs of the students in my classroom.

I appreciate the opportunities I had during this class that allowed me to see my own mathematical process. Realizing that many students need to be presented with opportunities that force this problem-solving process on them influenced the effort put into this unit. Continuing to strive to provide these opportunities for students in order to meet the needs of all students is and will remain a goal of mine in the years to come.

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