Lesson Plan – (Pre-Calculus or Calculus) – (Intro to area under the curve)
Note: Please attach any handouts that would be given to students and/or make it clear what problem(s) students will be working

Objective(s):	Students will gain understanding of finding area for different functions bounded by the x-axis and an interval. Students will construct an algorithm for the area model for a monotone increasing function with varying slope.
Grade Level OR Course Name	Pre-Calculus or Calculus (This could also be adapted to work for geometry.)
Est. Time	2 days at 45 minute class periods or 1 day at 90 minute class periods
Pre-requisite Knowledge:	Graphing by a given function: circles, square roots, absolute value, cubic functions, and piecewise functions. Find area of triangles, circles, rectangles, and trapezoids. Know how to generate a summation and use summation notation. Understand velocity, acceleration, and position as well as how all three are related. Know the formula d = r x t and be able to correctly calculate and analyze data corresponding to the formula relationship. Students need to know how to find limits to infinity and derivatives. Students need to know what a monotone increasing function with varying slope looks like graphically.
Vocabulary:	Area under a curve, Riemann Sums, algorithm, under estimate, over estimate, right end points, left end points
Materials Needed:	String, rulers, compass, protractor, POD Sheet, TEAM sheet, transparency grid paper, computers with internet access and the applet found at the following website: http://calculusapplets.com/riemann.html
Iowa Common Core Content Standards	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **(N-Q.1.)** 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **(A-CED.2.)** 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **(A-CED.3.)** 4. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **(F-IF.2.)** (Note: As a calculus lesson, it was difficult to find standards in the ICC that fit.)
Iowa Standards for Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Model with mathematics. 3. Use appropriate tools strategically. 4. Attend to precision. 5. Look for and make use of structure. 6. Look for and express regularity in repeated reasoning.

<u>Launch</u> (How will you engage students in the content for the day (or lesson)?)

When students enter the classroom they will be instructed to pick up the POD (Problem of the Day) sheet. (See POD sheet at the end of the lesson plan.) Protractors, compasses, rulers, and string will be available for students to use if the choose to. The purpose of this is to get them thinking about how difficult it is to calculate area when there is no grid present and the curve is not made of straight lines.

Explore (How will students explore the content for the day (or lesson)?) Reminder: focus on strategies to engage all students and to promote purposeful discussion amongst all students. How will you implement this lesson to help produce "patient problem solvers"?

Students will work in TEAMs of 3 on the TEAM sheet. TEAM sheet can be found at the end of the lesson. (There are two different types of collaborative work in my classes. One is titled "Group Work" and one is titled "TEAM". Group Work is used very rarely and is more for splitting up work to get a mundane task done in a more efficient manner. TEAM work is done on a regular basis and has many rules that are established from day one. Students know what is expected of them and how to work as a team. Some of the basics of TEAM work are: Everyone has a voice and needs to be heard. No one works alone and no one moves on until all parties understand what is going on. All TEAM members are responsible for learning and communicating. Each TEAM member must be able to explain solution steps and solutions to other groups or the whole class. There are many more things involved but these are the basics.) During TEAM work, I serve as facilitator and students think of me as a coach. I cannot do it for them but I can challenge them and guide them to complete it on their own.

As a whole class we will have a discussion about the problem of the day. What were the challenges, what would make it easier, etc. Students will get into their TEAMs and pick up the TEAM sheet. They will work through numbers 1-7 as a TEAM. I will facilitate and use guiding questions as necessary.

As a whole class we will discuss what they found on numbers 1-7. I will choose TEAMs to share out what they had for different numbers. I will make this decision based on what I see as I facilitate.

Students will log into computers and go to the website http://calculusapplets.com/riemann.html.
Each student will have their own computer and will go through the applet directions. They will still be in their TEAMs and asked to communicate with one another as they go. As a whole class we will discuss what they found working through the questions and manipulating the applet.

Students will work in their TEAMs again on question 9 and 10. They will share their algorithms by exchanging with another TEAM. They will need to give an example of a monotone increasing function for the other TEAM to use. Each TEAM will follow another TEAM's algorithm for number 9 with the function that was given to them. They will be asked to determine if they believe the given algorithm will work for all increasing monotone functions with a varying slope or not and why. They will also be asked to think of similarities and differences of the one they execute compared to the one they created. As a whole class we will share these findings.

Students will be asked to work on 11 and 12 on their own. After 5 minutes or so, they will be asked to work with their TEAM members on these two problems. As a whole class we will look at these. I will choose 2 to 3 TEAMs to share their solution method for problem 11 and a different set of 2 to 3 TEAMs to share their solution method for problem 12. Once the 4 to 6 solution methods are on the board, students will be asked to look at each for 1 minute with pencils down. Then they will be asked to write about similarities and differences in the different solutions for 2 minutes. With a row partner they will share what they wrote. As a whole group we will discuss their findings.

Summary/Close of the lesson (How will you close your lesson and bring student understanding to a close for the day?)

To close the lesson, I will briefly connect the idea of writing these areas as summations. We will talk about how this might look. I will guide them in developing the idea of rectangles and how their heights are related to the f(x) values and how we can choose the width. With my own applet at Stewart Calculus (part of my curriculum) I will show them how picking smaller values for change in x gets more rectangles making a better approximation.

Students will do a ticket out the door. The prompt will be, "How would you explain to a student who was absent today what we did in class? Give more than just the concluding argument."

Extension(s)

The purpose of this lesson is simply to get students feet wet in the idea of area under a curve being a summation of smaller figures that we know how to find the area of. Depending on how well students grasp this concept on day 1, an extension could be to ask them to generate a summation formula for this. (This is something that we will be working towards in the next couple of days.)

Check for Understanding (How will you assess students throughout and at the end of the lesson?)

While students are working on the different problems, I be checking in with them and asking questions. We will use thumbs up/side/down. As they share out I will ask questions for clarity. I will review the ticket out the door and the work they did on the problems during the class.

Key ideas/important points	Teacher strategies/actions
What it means to find an area bounded by a function over an interval.	Students will use the TEAM sheet and collaborate with one another to determine the area under different curves. I will facilitate the discussions asking guiding questions as necessary. As a whole group we will determine a more concrete procedure.
If we cannot find an exact area, we can get a good estimate. These are either over estimates or under estimates of the area.	I have given two graphs that the exact area can be found and two that the exact area is not as easy to find. Students will work with these on the TEAM sheet and compare and contrast the different functions analyzing whether the answers they obtained were exact, over estimates, or underestimates.
Exploring what it looks like to us rectangles of height f(x) and width of change of x to represent a good estimate for the area under a curve.	Students will use the applet provided to get a feel for what this looks like. They will be encouraged to combine the TEAM sheet and the ideas generated from the applet to solidify this idea. Again, we will share out. I will also use an applet from the Stewart Calculus website that goes with my curriculum to show a more in depth view of the rectangles than the other applet does.
Generalizing of this process by creating an algorithm.	I will be asking guiding questions for this as well as encourage students to connect number 6 and what we did with the applets.
Looking at applications for what finding the area under a curve tells us.	I will have students do number 11 and 12 of the TEAM sheet. I will facilitate this while they work through these encouraging them to think about the area in both cases as the accumulation of distance traveled. I will point out how the accumulation graph would look a little different for each as one is constant velocity and the other is varying velocity.

Key Ideas

Guiding Questions (focus on the mathematics and using open-ended questioning)

Good questions to ask	Possible student responses or actions	Possible teacher responses
How do you know if you can find an exact area?	If I can cut the shape up into rectangles, triangles, trapezoids, and sectors, I know I can find the exact area because they all have a formula that I know.	Do you think there is a way to find the exact area of any continuous curve? What do you think that might look like? What shape or shapes of the ones you listed closely resemble a portion of the graph in number 6?
How do you determine if your estimate is an over estimate or an underestimate?	If I covered more than the area it is an over estimate and if there was area left over that wasn't covered by my estimate, it is an under estimate.	What do these estimates tell you about the exact area of the curve? Is there a way to get a better estimate than the one you found? How could we use the over estimate and the under estimate of a function to get an even better estimate of the area?
How was finding the area in numbers 5 and 6 different from finding the areas in 3 and 4?	Numbers 3 and 4 were easier to find the area once I graphed them because they were shapes I know the area formula for. Number 5 and 6 I had to piece together to try to estimate the area.	How could you break the graphs up in numbers 5 and 6 to resemble a shape that you know the area formula for? Would this work for all continuous graphs? How could we find something that works for all continuous graphs?
How is using the graph for number 12 to find the area under the curve related to using the distance = rate x time formula?	Common answer would probably be, "I am not completely sure."	I would encourage students. I would provide guiding questions to help them arrive at this conclusion. These would include asking students if a table of time to velocity (rate) would help them construct a graph. If students have trouble with creating a table, I would help them do a simulation of the scenario.
How did using the applet help you determine how to generate the algorithm you did?	I could see how rectangles worked for every graph.	How did changing the width of the rectangles change the area? What width do you think is best to get a really good approximation? (If they say infinite, I would ask them to think about how a limit would relate to this idea and how we might take one.)

Misconceptions, Errors, Trouble Spots (Minimum 3 – focus on the mathematics)

Possible errors or trouble spots	Teacher questions/actions to resolve them
Students may have a difficult time starting the problem of the day because it is so vague.	I would ask them what they understood and what they needed to find. I would encourage them to explore all their resources including using the transparency grid to aid in counting.
Students may not make the connection that we can estimate area by using the same type of shape with the same width. For instance, on the absolute value graph, students may only see the two triangles and not think about using trapezoids of equal heights.	I would ask students if they could find the area in a different way. I would encourage them to look for a common shape that could be used repeatedly. If students are still not seeing it, I would have them draw a piecewise function that involves a rectangle and a triangle (linear function and constant function). I would help them break this up into other shapes with a common dimension.
Students may have a difficult time writing the algorithm.	I would ask students to explain to me in words how they found the area using the applet. As they explain this, I would help them connect their steps and trouble shoot with them to make sure that the algorithm flows. Having them use the graph from number 6 as a starting point would be encouraged.
Students may struggle with number 11 and 12.	I would encourage them to show the graph as an accumulation of distance traveled. If they are really struggling, I would have them do a simulation of the scenario given. We would chart the values from the simulation in a table with time vs velocity vs distance and discuss what two columns we need to graph based on the question asked. I would then ask how the other column is related to the graph or shown in the graph.

Name ______________________________
Date ____________ Period _____

Riemann Sums and Area Introduction

Problem of the Day:

Estimate the area of the shaded region below. Explain or show on the diagram how you found this area.

Is it possible to find the exact area of the shaded figure? Please explain why or why not?

Name ______________________________
Date ____________ Period _____

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Is it possible to find the exact area of the shaded figure? Please explain why or why not?

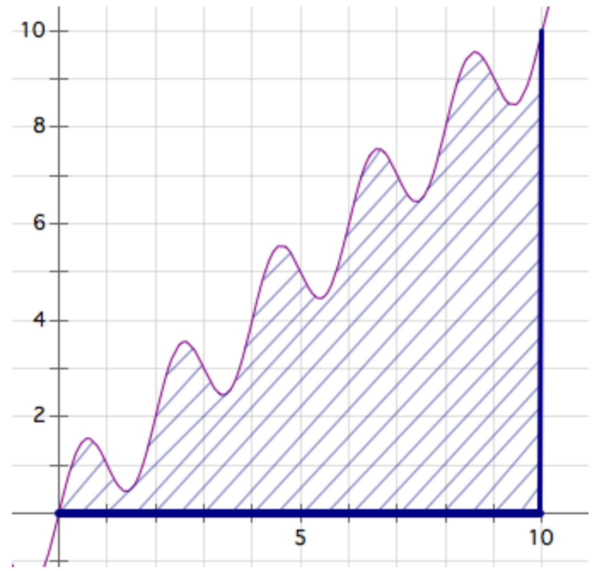
Name ____________________________
Date _______________ Period _____

Riemann Sums and Area Introduction

TEAM activity:

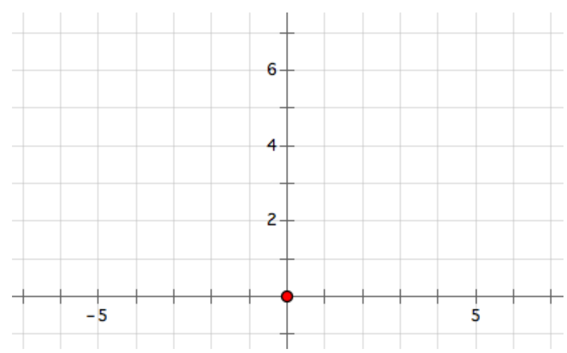
1. Discuss the problem of the day with your TEAM members.

2. As a TEAM determine a way to find a very close estimate to the area of the shaded region. Do you believe this estimate is an over estimate or an underestimate? Explain.

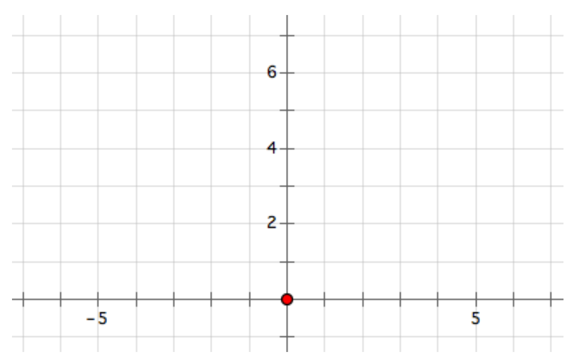


For numbers 3 – 6, find the area of the region bounded by the x-axis for the function given over the given interval. Explain whether or not you were able to find the exact area and why.

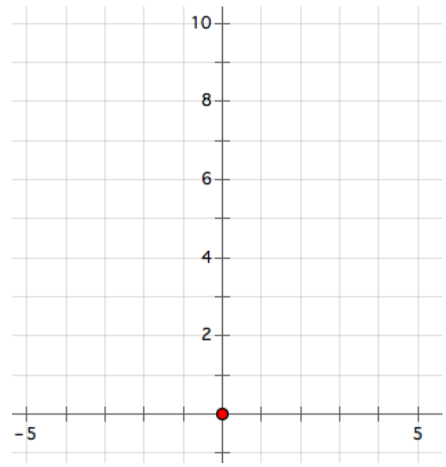
3. $f(x) = |x|$ on the interval [-6, 6].



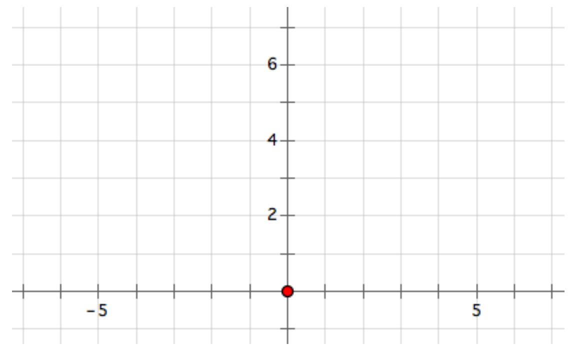
4. $f(x) = \sqrt{16 - x^2}$ on the interval [-4, 4]



5. $f(x) = x^2$ on the interval [-3, 3]



6. $f(x) = \sqrt{x+3}$ on the interval [-3, 5]



7. Looking back to numbers 3 – 6, did you find some easier than others? If so why? What were some similarities and differences you noticed in finding the area for each?

8. Go to http://calculusapplets.com/riemann.html and play with the applet midway down on the page. Follow the six instructions under “Try Following:”. Record your findings.

9. Create an algorithm for a monotone increasing function with a varying slope that is an underestimate.

10. How would you change your algorithm for a monotone increasing function with a varying slope that is an over-estimate.

Why do we find the area under a curve?

11. Given distance = rate x time. You are walking at a constant rate of 2 yards per second. How far do you travel in one minute? Represent this with a graph. Explain how the area of the graph is related to your total distance.

12. A RAGBRAI rider leaves his campsite and accelerates at 2 feet per second for 8 seconds. The rider then travels at a constant speed v(t) for 100 seconds. He has to brake and come to a complete stop at a check-point by decelerating at a rate of 4 feet per second.

    a. Find v(t).

    b. What is the distance traveled of the cyclist?